MICROWAVE BRIDGES

VANCE HOBERT HUDGINS

Library
U. S. Naval Postgraduate School
Monterey, California

Mont 225
8854

Library
U. S. Naval Postgraduate School
Annapolis, Md.

MICROWAVE BRIDGES

by

VANCE HOBERT HUDGINS

An Essay

Submitted to The Advisory Board of

The School of Engineering

of

The Johns Hopkins University

in conformity with the requirements for the

Degree of Master of Science in Engineering

Baltimore Maryland

May 1950

Thesis
4447

ACKNOWLEDGEMENT

The author wishes to express his gratitude to Dr. J. V. Lebacqz of The Johns Hopkins University for suggesting this topic and for his assistance and cooperation throughout the course of the investigation. Appreciation is also expressed to Dr. C. F. Miller and to the other members of the faculty and the department who aided considerably in this work.

13248

HOSEA

TABLE OF CONTENTS

1. SUMMARY 1

2. INTRODUCTION 2

3. PROPAGATION 4

4. HYBRID DEVICES 7

5. MAGIC TEE 9

   a. Basic Theory 9

   b. Operating Procedure for Measuring VSWR 17

   c. Design Considerations 18

   d. Relative Merits of the Tee Bridge and the Directional Coupler 19

   e. Use of the Tee Bridge About A Point Other than Match 20

   f. Measurement of Phase 20

6. HYBRID RING OR "RAT-RACES" 22

   a. Design Considerations 30

   b. Frequency Characteristics 30

7. APPLICATIONS 32

8. SIX-ARM MICROWAVE BRIDGE 38

   a. Design Considerations and Bridge Characteristics 42

   b. Four Terminal Application 45

9. CONCLUSION 46

10. APPENDIX A 47

11. APPENDIX B 54

12. APPENDIX C 65

13. BIBLIOGRAPHY 68

14. VITA 71

1. Preface 3
2. Introduction 3
3. Objectives 4
4. Problem Statement 5
5. Study Area 6
6. Study Design 7
7. Questionnaire Development and Administration 11
8. Sampling Procedure 13
9. Data Collection 15
10. Data Analysis 17
11. Results 19
12. Discussion 20
13. Conclusion 21
14. Recommendations 22
15. Limitations of the Study 23
16. Bibliography 24
17. Appendix 1 25
18. Appendix 2 26
19. Appendix 3 27
20. Appendix 4 28
21. Appendix 5 29
22. Appendix 6 30
23. Appendix 7 31
24. Appendix 8 32
25. Appendix 9 33
26. Appendix 10 34
27. Appendix 11 35
28. Appendix 12 36
29. Appendix 13 37
30. Appendix 14 38
31. Appendix 15 39
32. Appendix 16 40
33. Appendix 17 41
34. Appendix 18 42
35. Appendix 19 43
36. Appendix 20 44
37. Appendix 21 45
38. Appendix 22 46
39. Appendix 23 47
40. Appendix 24 48
41. Appendix 25 49
42. Appendix 26 50
43. Appendix 27 51
44. Appendix 28 52
45. Appendix 29 53
46. Appendix 30 54
47. Appendix 31 55
48. Appendix 32 56
49. Appendix 33 57
50. Appendix 34 58
51. Appendix 35 59
52. Appendix 36 60
53. Appendix 37 61
54. Appendix 38 62
55. Appendix 39 63
56. Appendix 40 64
57. Appendix 41 65
58. Appendix 42 66
59. Appendix 43 67
60. Appendix 44 68
61. Appendix 45 69
62. Appendix 46 70
63. Appendix 47 71
64. Appendix 48 72
65. Appendix 49 73
66. Appendix 50 74
67. Appendix 51 75
68. Appendix 52 76
69. Appendix 53 77
70. Appendix 54 78
71. Appendix 55 79
72. Appendix 56 80
73. Appendix 57 81
74. Appendix 58 82
75. Appendix 59 83
76. Appendix 60 84
77. Appendix 61 85
78. Appendix 62 86
79. Appendix 63 87
80. Appendix 64 88
81. Appendix 65 89
82. Appendix 66 90
83. Appendix 67 91
84. Appendix 68 92
85. Appendix 69 93
86. Appendix 70 94
87. Appendix 71 95
88. Appendix 72 96
89. Appendix 73 97
90. Appendix 74 98
91. Appendix 75 99
92. Appendix 76 100
93. Appendix 77 101
94. Appendix 78 102
95. Appendix 79 103
96. Appendix 80 104
97. Appendix 81 105
98. Appendix 82 106
99. Appendix 83 107
100. Appendix 84 108
101. Appendix 85 109
102. Appendix 86 110
103. Appendix 87 111
104. Appendix 88 112
105. Appendix 89 113
106. Appendix 90 114
107. Appendix 91 115
108. Appendix 92 116
109. Appendix 93 117
110. Appendix 94 118
111. Appendix 95 119
112. Appendix 96 120
113. Appendix 97 121
114. Appendix 98 122
115. Appendix 99 123
116. Appendix 100 124
117. Appendix 101 125
118. Appendix 102 126
119. Appendix 103 127
120. Appendix 104 128
121. Appendix 105 129
122. Appendix 106 130
123. Appendix 107 131
124. Appendix 108 132
125. Appendix 109 133
126. Appendix 110 134
127. Appendix 111 135
128. Appendix 112 136
129. Appendix 113 137
130. Appendix 114 138
131. Appendix 115 139
132. Appendix 116 140
133. Appendix 117 141
134. Appendix 118 142
135. Appendix 119 143
136. Appendix 120 144
137. Appendix 121 145
138. Appendix 122 146
139. Appendix 123 147
140. Appendix 124 148
141. Appendix 125 149
142. Appendix 126 150
143. Appendix 127 151
144. Appendix 128 152
145. Appendix 129 153
146. Appendix 130 154
147. Appendix 131 155
148. Appendix 132 156
149. Appendix 133 157
150. Appendix 134 158
151. Appendix 135 159
152. Appendix 136 160
153. Appendix 137 161
154. Appendix 138 162
155. Appendix 139 163
156. Appendix 140 164
157. Appendix 141 165
158. Appendix 142 166
159. Appendix 143 167
160. Appendix 144 168
161. Appendix 145 169
162. Appendix 146 170
163. Appendix 147 171
164. Appendix 148 172
165. Appendix 149 173
166. Appendix 150 174
167. Appendix 151 175
168. Appendix 152 176
169. Appendix 153 177
170. Appendix 154 178
171. Appendix 155 179
172. Appendix 156 180
173. Appendix 157 181
174. Appendix 158 182
175. Appendix 159 183
176. Appendix 160 184
177. Appendix 161 185
178. Appendix 162 186
179. Appendix 163 187
180. Appendix 164 188
181. Appendix 165 189
182. Appendix 166 190
183. Appendix 167 191
184. Appendix 168 192
185. Appendix 169 193
186. Appendix 170 194
187. Appendix 171 195
188. Appendix 172 196
189. Appendix 173 197
190. Appendix 174 198
191. Appendix 175 199
192. Appendix 176 200
193. Appendix 177 201
194. Appendix 178 202
195. Appendix 179 203
196. Appendix 180 204
197. Appendix 181 205
198. Appendix 182 206
199. Appendix 183 207
200. Appendix 184 208
201. Appendix 185 209
202. Appendix 186 210
203. Appendix 187 211
204. Appendix 188 212
205. Appendix 189 213
206. Appendix 190 214
207. Appendix 191 215
208. Appendix 192 216
209. Appendix 193 217
210. Appendix 194 218
211. Appendix 195 219
212. Appendix 196 220
213. Appendix 197 221
214. Appendix 198 222
215. Appendix 199 223
216. Appendix 200 224
217. Appendix 201 225
218. Appendix 202 226
219. Appendix 203 227
220. Appendix 204 228
221. Appendix 205 229
222. Appendix 206 230
223. Appendix 207 231
224. Appendix 208 232
225. Appendix 209 233
226. Appendix 210 234
227. Appendix 211 235
228. Appendix 212 236
229. Appendix 213 237
230. Appendix 214 238
231. Appendix 215 239
232. Appendix 216 240
233. Appendix 217 241
234. Appendix 218 242
235. Appendix 219 243
236. Appendix 220 244
237. Appendix 221 245
238. Appendix 222 246
239. Appendix 223 247
240. Appendix 224 248
241. Appendix 225 249
242. Appendix 226 250
243. Appendix 227 251
244. Appendix 228 252
245. Appendix 229 253
246. Appendix 230 254
247. Appendix 231 255
248. Appendix 232 256
249. Appendix 233 257
250. Appendix 234 258
251. Appendix 235 259
252. Appendix 236 260
253. Appendix 237 261
254. Appendix 238 262
255. Appendix 239 263
256. Appendix 240 264
257. Appendix 241 265
258. Appendix 242 266
259. Appendix 243 267
260. Appendix 244 268
261. Appendix 245 269
262. Appendix 246 270
263. Appendix 247 271
264. Appendix 248 272
265. Appendix 249 273
266. Appendix 250 274
267. Appendix 251 275
268. Appendix 252 276
269. Appendix 253 277
270. Appendix 254 278
271. Appendix 255 279
272. Appendix 256 280
273. Appendix 257 281
274. Appendix 258 282
275. Appendix 259 283
276. Appendix 260 284
277. Appendix 261 285
278. Appendix 262 286
279. Appendix 263 287
280. Appendix 264 288
281. Appendix 265 289
282. Appendix 266 290
283. Appendix 267 291
284. Appendix 268 292
285. Appendix 269 293
286. Appendix 270 294
287. Appendix 271 295
288. Appendix 272 296
289. Appendix 273 297
290. Appendix 274 298
291. Appendix 275 299
292. Appendix 276 300
293. Appendix 277 301
294. Appendix 278 302
295. Appendix 279 303
296. Appendix 280 304
297. Appendix 281 305
298. Appendix 282 306
299. Appendix 283 307
300. Appendix 284 308
301. Appendix 285 309
302. Appendix 286 310
303. Appendix 287 311
304. Appendix 288 312
305. Appendix 289 313
306. Appendix 290 314
307. Appendix 291 315
308. Appendix 292 316
309. Appendix 293 317
310. Appendix 294 318
311. Appendix 295 319
312. Appendix 296 320
313. Appendix 297 321
314. Appendix 298 322
315. Appendix 299 323
316. Appendix 300 324
317. Appendix 301 325
318. Appendix 302 326
319. Appendix 303 327
320. Appendix 304 328
321. Appendix 305 329
322. Appendix 306 330
323. Appendix 307 331
324. Appendix 308 332
325. Appendix 309 333
326. Appendix 310 334
327. Appendix 311 335
328. Appendix 312 336
329. Appendix 313 337
330. Appendix 314 338
331. Appendix 315 339
332. Appendix 316 340
333. Appendix 317 341
334. Appendix 318 342
335. Appendix 319 343
336. Appendix 320 344
337. Appendix 321 345
338. Appendix 322 346
339. Appendix 323 347
340. Appendix 324 348
341. Appendix 325 349
342. Appendix 326 350
343. Appendix 327 351
344. Appendix 328 352
345. Appendix 329 353
346. Appendix 330 354
347. Appendix 331 355
348. Appendix 332 356
349. Appendix 333 357
350. Appendix 334 358
351. Appendix 335 359
352. Appendix 336 360
353. Appendix 337 361
354. Appendix 338 362
355. Appendix 339 363
356. Appendix 340 364
357. Appendix 341 365
358. Appendix 342 366
359. Appendix 343 367
360. Appendix 344 368
361. Appendix 345 369
362. Appendix 346 370
363. Appendix 347 371
364. Appendix 348 372
365. Appendix 349 373
366. Appendix 350 374
367. Appendix 351 375
368. Appendix 352 376
369. Appendix 353 377
370. Appendix 354 378
371. Appendix 355 379
372. Appendix 356 380
373. Appendix 357 381
374. Appendix 358 382
375. Appendix 359 383
376. Appendix 360 384
377. Appendix 361 385
378. Appendix 362 386
379. Appendix 363 387
380. Appendix 364 388
381. Appendix 365 389
382. Appendix 366 390
383. Appendix 367 391
384. Appendix 368 392
385. Appendix 369 393
386. Appendix 370 394
387. Appendix 371 395
388. Appendix 372 396
389. Appendix 373 397
390. Appendix 374 398
391. Appendix 375 399
392. Appendix 376 400
393. Appendix 377 401
394. Appendix 378 402
395. Appendix 379 403
396. Appendix 380 404
397. Appendix 381 405
398. Appendix 382 406
399. Appendix 383 407
400. Appendix 384 408
401. Appendix 385 409
402. Appendix 386 410
403. Appendix 387 411
404. Appendix 388 412
405. Appendix 389 413
406. Appendix 390 414
407. Appendix 391 415
408. Appendix 392 416
409. Appendix 393 417
410. Appendix 394 418
411. Appendix 395 419
412. Appendix 396 420
413. Appendix 397 421
414. Appendix 398 422
415. Appendix 399 423
416. Appendix 400 424
417. Appendix 401 425
418. Appendix 402 426
419. Appendix 403 427
420. Appendix 404 428
421. Appendix 405 429
422. Appendix 406 430
423. Appendix 407 431
424. Appendix 408 432
425. Appendix 409 433
426. Appendix 410 434
427. Appendix 411 435
428. Appendix 412 436
429. Appendix 413 437
430. Appendix 414 438
431. Appendix 415 439
432. Appendix 416 440
433. Appendix 417 441
434. Appendix 418 442
435. Appendix 419 443
436. Appendix 420 444
437. Appendix 421 445
438. Appendix 422 446
439. Appendix 423 447
440. Appendix 424 448
441. Appendix 425 449
442. Appendix 426 450
443. Appendix 427 451
444. Appendix 428 452
445. Appendix 429 453
446. Appendix 430 454
447. Appendix 431 455
448. Appendix 432 456
449. Appendix 433 457
450. Appendix 434 458
451. Appendix 435 459
452. Appendix 436 460
453. Appendix 437 461
454. Appendix 438 462
455. Appendix 439 463
456. Appendix 440 464
457. Appendix 441 465
458. Appendix 442 466
459. Appendix 443 467
460. Appendix 444 468
461. Appendix 445 469
462. Appendix 446 470
463. Appendix 447 471
464. Appendix 448 472
465. Appendix 449 473
466. Appendix 450 474
467. Appendix 451 475
468. Appendix 452 476
469. Appendix 453 477
470. Appendix 454 478
471. Appendix 455 479
472. Appendix 456 480
473. Appendix 457 481
474. Appendix 458 482
475. Appendix 459 483
476. Appendix 460 484
477. Appendix 461 485
478. Appendix 462 486
479. Appendix 463 487
480. Appendix 464 488
481. Appendix 465 489
482. Appendix 466 490
483. Appendix 467 491
484. Appendix 468 492
485. Appendix 469 493
486. Appendix 470 494
487. Appendix 471 495
488. Appendix 472 496
489. Appendix 473 497
490. Appendix 474 498
491. Appendix 475 499
492. Appendix 476 500
493. Appendix 477 501
494. Appendix 478 502
495. Appendix 479 503
496. Appendix 480 504
497. Appendix 481 505
498. Appendix 482 506
499. Appendix 483 507
500. Appendix 484 508
501. Appendix 485 509
502. Appendix 486 510
503. Appendix 487 511
504. Appendix 488 512
505. Appendix 489 513
506. Appendix 490 514
507. Appendix 491 515
508. Appendix 492 516
509. Appendix 493 517
510. Appendix 494 518
511. Appendix 495 519
512. Appendix 496 520
513. Appendix 497 521
514. Appendix 498 522
515. Appendix 499 523
516. Appendix 500 524

SUMMARY

This paper reviews and discusses the fundamental behavior of several microwave bridges. Some basic theory and design consideration is given. This paper is qualitative in nature with some mathematical analysis given in the appendices. The propagation characteristics in wave-guides and the phase properties of simple wave-guide tee junctions are briefly considered. Two kinds of hybrid circuits are then described, the one involving a ring or loop of transmission lines, the other relying upon the symmetry properties of certain four-arm junctions. The description is centered about wave-guide structures for microwaves, but the principles may also be applied to other kinds of transmission lines for other frequency ranges. The relative merits of directional couplers and tee bridge elements are discussed. A wave-guide structure, which is a true microwave equivalent of a Wheatstone bridge, is discussed in detail.

Some of the more important applications are outlined and a discussion of the merits of each bridge is given.

INTRODUCTION

The wave-guide has been developed in the last fifteen years into a very important medium for microwave transmission. During the war progress was especially rapid due to the use of wave-guides in radar. Many of the most important developments in radar would have been virtually impossible without the use of these guides. It is now possible to foresee some of the probable uses of wave-guides in communications developments of the near future. Hollow-pipe guides are not practical until the wavelength is shorter than about thirty centimeters. At this limit both coaxial cables and wave-guides are used. As the wavelengths get shorter, the wave-guide becomes increasingly attractive until at ten centimeters it is preferred for efficient, shielded transmission of power, and at three centimeters it is used almost exclusively.

Although the earliest work emphasized the wave-guide as a form of transmission line for radio waves, it was apparent from the outset that wave-guide structures would be required also to serve as numerous basic electrical components. These components at low frequencies include such elements as resistors, transformers, relays, switches, coils, capacitors, etc. Wave-guide counterparts of these low frequency components have been constructed to operate in the microwave region. This paper presents a discussion of one group of these components called "Microwave Bridges."

The behavior of microwave bridges is similar to that of the conventional low-frequency bridge circuits which compare the load impedances of the collinear arms, the signal being fed in one branch arm and a detector on the other. A microwave bridge consists of three groups of components: r-f sources, the bridge element, and the detector-indicator. Of these, the bridge element is the most interesting since it must couple to the detector only that power which is reflected from the test piece. Therefore, most of this paper is concerned with a discussion of the bridge elements. The electrical behavior, physical form, possible applications, and other characteristics of several such bridge elements will be discussed.

PROPAGATION

It is well known that radio waves can be freely propagated within any conducting tube, so long as the wavelength is shorter than a cutoff wavelength determined by the cross sectional size and shape of the pipe. There are many transmission modes, corresponding to different patterns of lines of force, which can be established and maintained for wave transmission, and thus many different cutoff wavelengths. However, for reasons of simplicity, wave-guide developments have so far been confined almost entirely to the mode of longest cutoff wavelength, the so-called dominant wave, in pipes so proportioned that no other mode can be freely sustained. Lines of electric intensity in the dominant wave are shown in Figure 1 for tubes of circular, square, and oblong cross section. There is evidently a close resemblance to radio waves. For practical reasons circular and rectangular shapes are in general use; however, other shapes are being introduced.

Patterns are shown in Figure 1 for both vertical and horizontal polarization. In the circular and square wave guides, the electric field configuration may have any orientation, depending upon the manner in which the waves were initially launched. However, in the rectangular guide the cutoff wavelength corresponding to the cross-polarized oscillation can be depressed below the operating wavelength, and only the fixed orientation shown can be freely transmitted.

Consequently, a rectangular wave guide is preferred whenever there is a chance that the polarization may be rotated in an uncontrolled fashion. Experience has shown that unavoidable irregularities, or deliberate distortions, such as bends, do tend to rotate the plane of polarization in circular wave guides to such an extent as to interfere seriously with the normal operation of the system. In general, therefore, wave guide components have been developed with rectangular guides. Best results are generally obtained when the proportions of the cross section lie in the vicinity of 1 by 2, with the larger inside dimension equal to about three-quarters of a wavelength. The microwave bridges discussed here are constructed with rectangular wave guides.

ELECTRIC FIELD CONFIGURATION IN CIRCULAR, SQUARE, AND OBLONG GUIDES

FIG. 1

HYBRID DEVICES

Several hybrid devices have been developed which perform many important functions at microwave frequencies including uncoupling the receiver from the transmitter, isolating the local beating oscillator from the antenna, and that of a bridge circuit.

The performance of these devices is very similar to that of the hybrid coil, long familiar in telephone practice as a means for securing preferential isolation of circuits.

A hybrid device is shown in Figure 2 schematically as an eight terminal network. When a source of alternating electromotive force is connected across the terminals A, no signal appears at the terminals C, but transmission takes place freely between A and B and between A and D, with the input thus divided between equal loads placed at B and D. If the generator is transferred from A to C, no voltage can appear at A, from reciprocity considerations and again the input power is divided in some fashion between loads at B and D. It is generally desirable, moreover, that the terminals B and D be balanced with respect to each other, so that a generator applied at either of these points delivers power only to loads at A and C. In most cases the circuit is so proportioned as to bring about equal power division between the driven loads.

In constructions suitable for use at microwave frequencies, the terminals correspond to four appropriate transmission lines, not necessarily alike, emerging from the network. The loads

or generators attached to these lines are assumed to terminate each arm in its characteristic impedance. If this is not done, reflections from the loads will tend to upset the balance.

The elements comprising the hybrid network are presumed to be primarily reactive, so that internal dissipation is reduced to a minimum. The resistive component of the impedance viewed at any one of the emerging lines corresponds, therefore, almost solely to the loads in the two adjacent lines, transformed by the network. A properly compensated circuit is one which combines these loads so as to present at any arm an impedance equal as nearly as possible to the characteristic impedance of that arm.

Within the hybrid network the four emerging lines must be joined in some manner. Two distinct possibilities can be visualized. The terminal lines may all be connected to another transmission line or they may be joined directly without any intervening medium. Correspondingly, two classes of circuits possessing hybrid characteristics will be described: hybrid junctions or "magic tees" as shown in Figure 3 and hybrid rings or "rat-races" involving a ring or loop of line to which connections are appropriately made.

MAGIC TEE

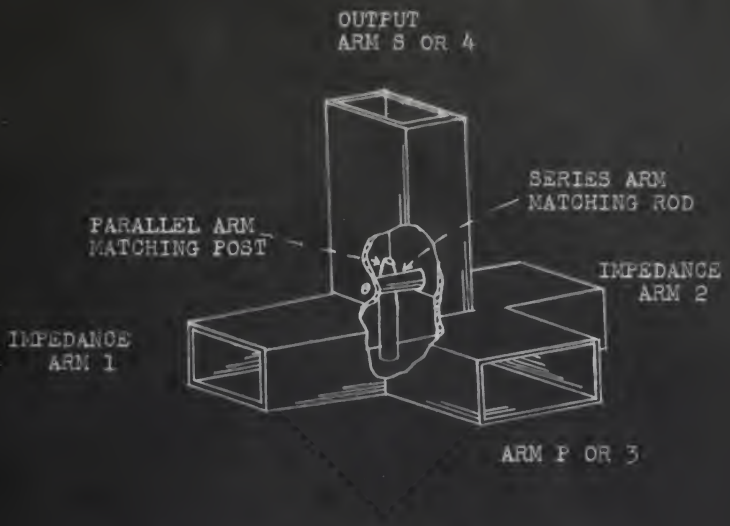
A hybrid network for microwaves can be secured in the form of a compound junction comprising a series connection and a parallel connection made to a guide at the same point on its longitudinal axis. Figure 3 shows a form of such a junction called the magic tee.

The fundamental property of the tee is that, if identical impedances are placed on the arms which are labelled impedance arms, then no power will be transmitted from the input to the output arm. This can be demonstrated by the use of symmetry arguments alone for the geometrical configuration shown in Figure 3 or by the analogy with optics. For a mathematical analysis see Appendix A.

This property suggests a method for comparing two impedances, since if the impedances are placed on the impedance arms and if there is no transmission through the tee, then the impedances are identical. When the impedances are not identical, the transmission through the tee gives a measure of the degree of difference of the impedances.

Basic Theory: The magic tee shown in Figure 3 is an example of the eight terminal network in Figure 2 and possesses the symmetry properties of that network.

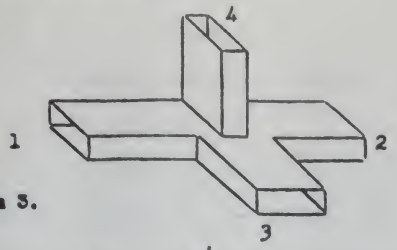
Some approximate field configurations are illustrated in Figure 4, which suggests how the geometric symmetry of the bridges produces the necessary electrical symmetries.



A PERSPECTIVE CUT-AWAY VIEW OF A MAGIC TEE USING WAVEGUIDE ARMS SHOWING "ROD" AND "POST" USED IN IMPEDANCE MATCHING.

FIG. 3

A bridge using waveguide arms



fed from Arm 3.

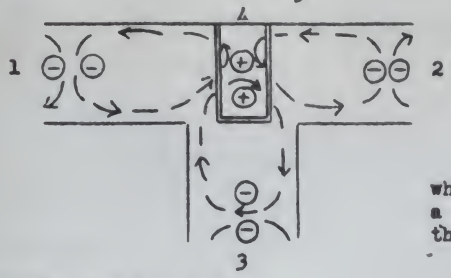
E ——→

H - - - →

⊖ into paper.

⊕ out of paper

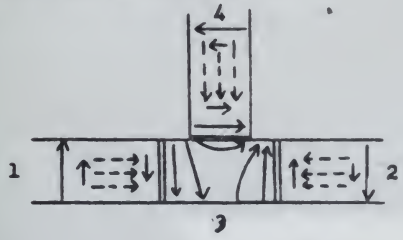
(a)



which cannot excite a wave in Arm 4 of the form:

and conversely, fed from Arm 4:

(b)



which cannot excite a wave in Arm 3 of the form:

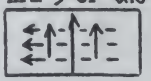


FIG. 4

Consider Figure 5 which shows the magic tee being used as a bridge to measure voltage standing wave ratio.

In general there will be waves traveling in both directions in all four arms of the tee. These waves will be described by their complex voltage amplitudes. Assume that the generator voltage is 1 + j0.

A double subscript will be used to designate voltage amplitudes. Thus E_{3T} is the voltage amplitude of the wave traveling from 3 into the tee. E_{T4} is the voltage amplitude of the wave traveling from the tee to 4. Since arm 2 is terminated with a matched load $E_{2T} = 0$.

Due to the symmetrical properties of the tee, if arm 1 is matched, there will be no power reaching 4 (the detector). However, if this arm is not matched, the reflected wave from 1 will cause power to be transmitted to the detector.

$E_4 = kE_{1T}$ where k is a function of frequency, and of the generator bridge, and detector impedance, but not of Z_1 (unknown impedance)

Let Z_d = impedance of the detector

R_d = real part of the Z_d

P_d = power to the detector

then:

$$P_d = \left|\frac{E_4}{Z_d}\right|^2 R_d = \left|\frac{kE_{1T}}{Z_d}\right|^2 R_d$$

In arm 1:

$E_{1T} = A_1 E_{T1}$ where A_1 is the voltage reflection coefficient

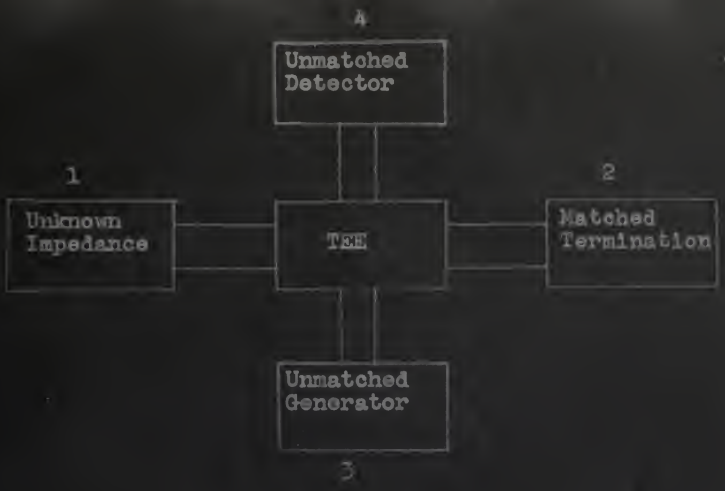


FIG. 5

Lines of electric intensity with a metal post

Circuit equivalent

Current flow with metal rod

Circuit equivalent

A "post" at upper left is equivalent to a shunt capacitor, while a "rod", lower left is equivalent to a shunt coil

FIG. 8

of the unknown impedance. Also, E_{T1} is the sum of contributions from two sources, the original wave in arm 1 from the generator (E_o) and a contribution (cE_{1T}) from the wave E_{1T}, part of which is reflected from the bridge.

i.e. $E_{T1} = E_o + cE_{1T}$, where c is the voltage reflection coefficient of the bridge looking in at arm 1 with all other terminals of the bridge loaded as shown in Figure 5.

Solving: $\frac{E_{1T}}{A_1} = E_o + cE_{1T}$

$$E_{1T} = \frac{A_1 E_o}{(1-A_1 c)} \quad \text{and}$$

$$(1) \quad P_d = c_2 |a_1|^2 \frac{1}{(1-A_1 c)^2}$$

The proportionality factor is usually determined experimentally in practice from (1)

$$A_1 = k\sqrt{P_d}\ |1 - A_1 c|$$

Now if $c = 0$ the power out is a true measure of A_1 or of the VSWR of the unknown impedance. Even if $c \neq 0$, the values of c and A_1 usually encountered do not give large deviations from the simple expressions. Figures 6 and 7 show the maximum error possible by neglecting the $|1 - A_1 c|$ term. This error is in the form of a phase error. At a given λ and bridge adjustment, c will be constant. If a_1 is constant in magnitude and varies in phase, the term $|1 - A_1 c|$ will vary about the value 1. There will be two phases of the unknown for which there will be no error.

In addition to this phase error, there is usually an

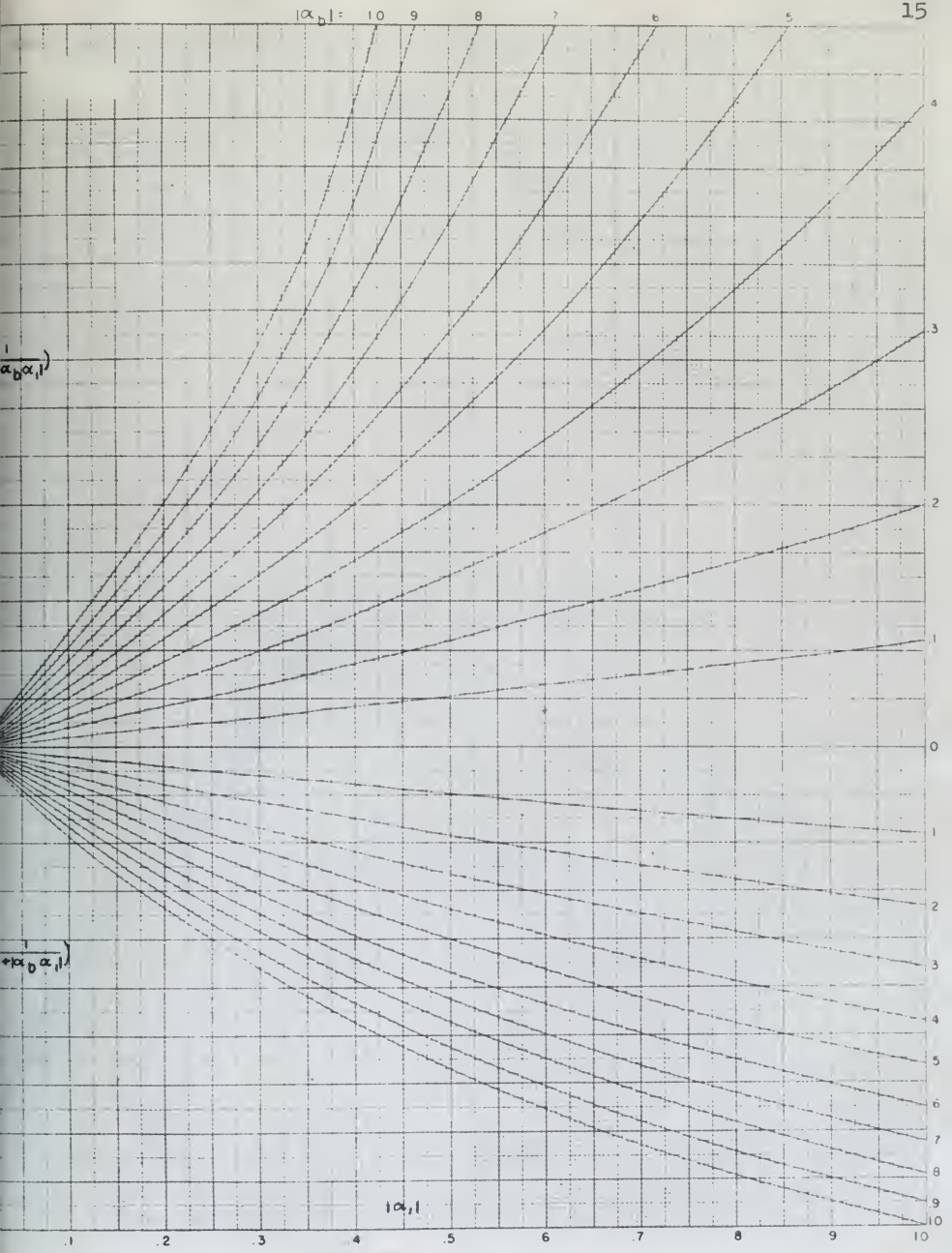

$|\alpha_b| =$ 10 9 8 7 6 5
4
3
2
1
0
1
2
3
4
5
6
7
8
9
10
$\frac{1}{\alpha_b \alpha_1}$
$\frac{1}{+|\alpha_b \alpha_1|}$
$|\alpha_1|$
.1 .2 .3 .4 .5 .6 .7 .8 .9 10

FIG. 6

13249

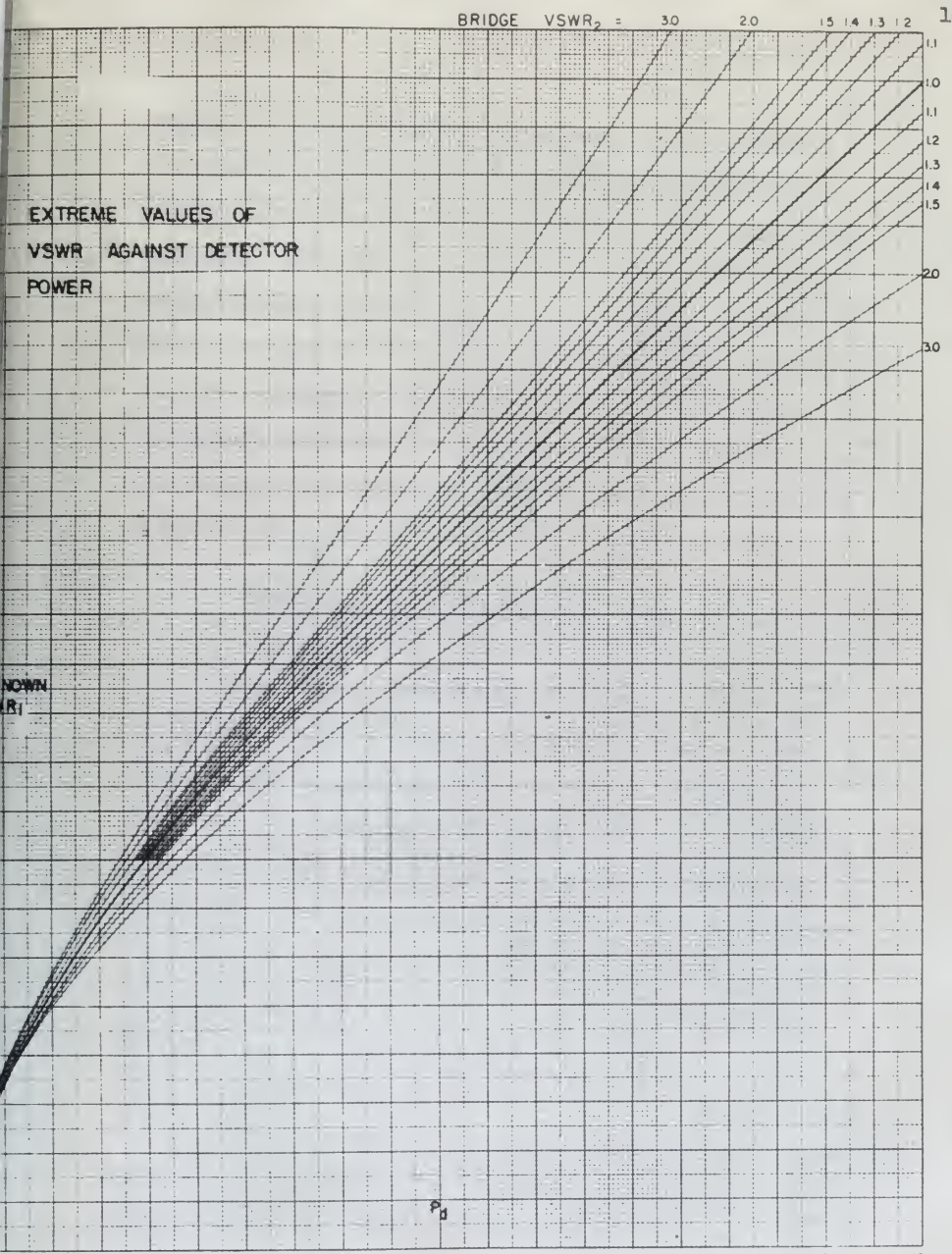

BRIDGE VSWR2 = 3.0 2.0 1.5 1.4 1.3 1.2
1.1
1.0
1.1
1.2
1.3
1.4
1.5
2.0
3.0
EXTREME VALUES OF
VSWR AGAINST DETECTOR
POWER
NOWN
R1
Pd
1
2
3
4
5
6
7
8
9
10

FIG. 7

error due to inherent unbalance of the tee, or, what amounts to the same thing, failure to use a perfectly matched termination on arm 2. This unbalance can be specified by giving the VSWR of the load on arm 1 which is necessary for balance. The error in measurement thus introduced behaves in the same way as a reflection in the line (of the same VSWR) in slotted section measurements.

For example, if a load of VSWR = 1.02 is required to balance the bridge, an unknown which apparently has a VSWR = ν as measured by the bridge, can actually have a VSWR lying between $\nu \times 1.02$ and $\frac{\nu}{1.02}$. This is in addition to any phase error.

Operating Procedure For Measuring VSWR: Usually the device to be tested must have a VSWR < N. The test equipment consists of the bridge element, a detector, a modulated CW oscillator, and an amplifier driving a meter. The test sample is placed on one impedance arm and a matched load on the other. To adjust the power level, a termination having a known VSWR = N is furnished. With this standard on the test arm, the level of the bridge is adjusted to give reference deflection on the meter. The test items can then be placed on the test arm in rapid succession and accepted or rejected. The setting of the bridge can readily be checked at any time against the standard mismatch.

The meter scale can be graduated to read directly in VSWR.

This procedure is of particular importance if it is necessary to tune the device tested to a match, as in the case of thermistor mounts. The device is adjusted while it is on the bridge,

and it is necessary only to obtain a minimum meter deflection.

The primary failing of this type of bridge is that it fails to give phase data. For many types of operations, however, this is no handicap.

Design Considerations: The features just described, depending upon the combination of geometrical symmetry with the polarization properties of rectangular wave guide, are inherent in the junction by itself. An important refinement is realized, however, when reactive tuning, as exemplified by the metal post and rod in Figure 3 is associated with the junction. These added elements improve the impedance match of the junction to each of the four arms, and thereby automatically bring about a high degree of balance between arms 1 and 2. Figure 8 shows the low frequency equivalents of the post and rod. The metal post concentrates the electric field between itself and the nearby wall of the guide, and is equivalent to a capacitance placed in shunt across a transmission line. The principal effect of the metal rod is to provide paths for conductive current flow between the connected walls of the guide, with a resulting increase in magnetic flux and flux linkages, and thus it is like an inductance in shunt across a line. Provided that reasonably good conductors have been selected for the post and rod, their resistance components can be neglected except in extreme cases.

Other tuning means, such as metal plates, can be used instead of the metal rod and post, but the later is being widely used.

By a proper choice of tuning elements, effective neutralization of reflections can be obtained over a remarkably broad band of frequencies.

In practice magic tees at X-band, can be built without tuning devices to be balanced at a VSWR of 1.01. Such tees should be constructed of special accurate wave guide, with equal impedance arms and identical line fittings. An estimated accuracy of ± .001" in the alignment of arms 3 and 4 is required.

Another important characteristic of a magic tee is the impedance looking into the test arm (reflection coefficient = c). Since c is a function of the generator and detector impedances, it can be readily adjusted by tuners in either of these arms. For a broad band solution, however, it is desirable to use a matched generator and detector, and tune c by means of tuners placed close to the center of the tee junction. This problem is being worked on but a completely satisfactory solution is not available. The c can be reduced by inserting a small well-matched pad in the test arm without invalidating the theory, but at a cost of a loss in sensitivity of twice the attenuation of the inserted pad. This trick may be useful in measuring large VSWR's.

Relative Merits of the Tee Bridge and the Directional Coupler: Directional couplers can satisfy the requirements of a bridge element and are being used in this manner at present. See Appendix C for the mathematical analysis. Therefore, the relative merits of the tee as a bridge and the directional coupler should

be discussed.

The magic tee has the advantage on balance (or what is called directivity in a directional coupler) since it depends on geometrical symmetry alone. Careful design is necessary to produce a coupler with high enough directivity (about 30 db) to be useful for low VSWR's. The coupler can be trimmed with tuning screws to be accurately balanced but the frequency sensitivity will not be good. The c (reflection coefficient) is much better in the case of the directional coupler, since if a padded generator is used, the VSWR looking back from the test impedance can be very good indeed. This becomes more important in measuring large VSWR's. The directional coupler subtracts only a small fraction of the power in the line, which may be either an advantage or a disadvantage. This results in a loss in sensitivity, but it permits measurements to be made on a unit while in service.

Use of the Tee Bridge About A Point Other Than Match: Sometimes it is desired that a device have a definite impedance other than the line impedance. For this case, instead of placing a match on one of the impedance arms, one uses as a standard a unit having the desired impedance. Then, when the test item has the same impedance, the bridge is balanced. Instead of measuring the VSWR of the unknown, the VSWR meter now indicates the degree of difference between the unknown and the standard.

Measurement of Phase: The tee bridge does not measure phase in the rapid automatic way that it measures VSWR, but there are ways of using the bridge to get phases which can compete with

slotted section methods. The tee bridge can be used with a calibrated variable impedance on one arm to make measurements, but the calibration and use of the variable impedance is laborious. This method is no more accurate than the method used to calibrate the variable impedance.

A subsidiary phase measuring device can be added to the tee bridge which consists of a sliding screw in the test arm. It is inserted to a fixed depth, and its position along the guide varied maxima and minima will be observed, on the indicating meter. These will be more pronounced than with a slotted section. A phase calibration can be made by placing an impedance of known phase on the test arm and observing the phase of the sliding screw. Unfortunately, if $c \neq 0$, there is a phase error, and the maxima and minima may not be evenly spaced. This is more pronounced at high VSWR. For very low VSWR this method may be preferable to the slotted section since the ratio of maximum to minimum will be large, especially if the sliding screw reflects about as much as the unknown. The sliding screw should be carefully built in a narrow slot. This screw is withdrawn completely while measuring VSWR.

HYBRID RINGS OR "RAT RACES"

An examination of Figure 9 will show the principles involved in the construction of a hybrid network from a ring or loop of transmission lines. The cross section shown is in the electric plane of a wave guide ring with a straight guide connected to it symmetrically from the exterior. The electrical length around the ring is one and one-half wave lengths. Here, wavelength corresponds to the increased phase-velocity characteristic of a hollow pipe. It is assumed, and has been verified experimentally, that the electrical length around the ring is equal to the mean circumference.

When dominant waves are sent into the ring from the side arm, they will be split at the junction into two sets of waves of equal amplitude traveling around the ring in opposite directions. A pure standing wave will therefore be set up within the ring. This must be the case for no mechanism has been provided for dissipation within the ring other than the minor ohmic losses associated with wave transmission. The two sets of waves spreading out from the junction are 180 degrees out of phase, due to the nature of an electric-plane tee junction. When they reach the point diametrically opposite the junction, they have traversed paths of equal length and are still 180 degrees out of phase. This point is therefore a voltage minimum (zero). With this as a starting point, the standing wave within the ring can be plotted by marking off alternate voltage maxima and minima at 1/4 wave

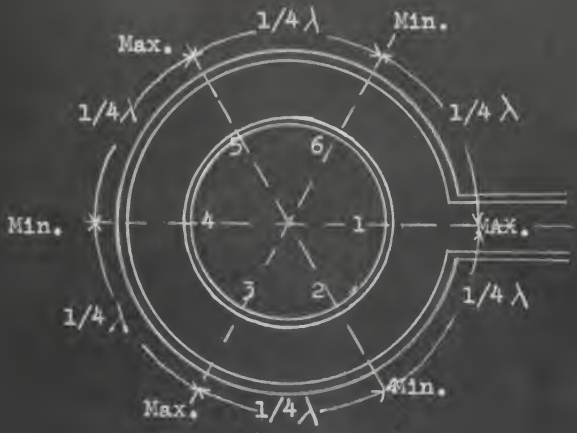

Max.
1/4 λ
Min.
1/4 λ
1/4 λ
Min.
Max.
1/4 λ
1/4 λ
Max.
1/4 λ
Min.
1
2
3
4
5
6

FIG. 9

3/4 λ
K₀
K₀
K₀
1/4 λ
1/4 λ
K₀
K₀
1/4 λ
K₀
1
2
3
4
5
6

FIG. 10

intervals.

If identical wave guides are connected symmetrically to the ring at points 2 and 4 so as to form additional electric-plane tee junctions, and if these side arms are terminated in their characteristic impedance , they will receive equal amounts of power which are large, since they are series connections at voltage minima. The amplitude of waves proceeding past 2 toward 3 will however still be equal to the amplitude of waves proceeding past 4 toward 3, since equal amounts of power are being extracted from two equal sets of waves. The arc 2 3 4 still contains, therefore, a pure standing wave. Since identical phase shifts have been introduced at 2 and 4 by the identical tee junctions, the position of this standing wave is unaltered, and there is still a voltage maximum at 3. A series connection to the ring at 3 will thus receive no power. Such a connection may be made by introducing a fourth electric plane junction at 3.

Figure 10 is a cross sectional view with series connections to the ring at 1, 2, 3, and 4. This represents a hybrid circuit with bidirectional characteristics. This may be shown by examining what happens when a generator is placed on arm 2 and appropriate loads in arms 1 and 3. At 5, the point diametrically opposite the driving point, the fields will interfere to produce a voltage minimum, and in the absence of arms 1, 3, and 4, there will be voltage maxima at 2, 4, and 6, and additional minima at 1 and 3. Arms 1 and 3 will thus receive equal amounts of power

without disturbing the position of the standing wave pattern in the arc 153 and the arm at 4 receives no power.

The ring described thus far is not the only structure of this type which functions as a hybrid circuit. If a series element of impedance Z on a transmission line of characteristic impedance K_0 is replaced by a shunt element of impedance $\frac{K_0^2}{Z}$ on one side of which is added a quarter wavelength of line and on the other side, a three-quarter wavelength of line, the entire performance of the circuit is unaltered. In this way the four series connections to the ring can be replaced, one by one, by shunt connections. This gives six distinct hybrid rings which are shown in Figure 11. These circuits apply equally well for low and high frequencies.

For microwave frequencies the practical realization of the circuits shown in Figure 11 calls for electric plane wave guide tee junctions as series connections and magnetic plane wave guide tee junctions or transformations to coaxial as parallel connections. In certain combinations there may be structural advantages in making the plane of the ring the magnetic plane. An interesting situation arises when square or circular wave guide is used, for here the circuits of Figure 11a and 11f can be included in the same structure. For one orientation of the polarization the ring lies in the electric plane; for another, the magnetic plane; and for any orientation in general, the input waves may be resolved into components parallel with and perpen-

(a)

(d)

(b)

(e)

(c)

(f)

FIG. 11

dicular to the plane of the ring for each of which a hybrid circuit is provided. In this case the matching of resistive impedances for both components becomes somewhat more involved.

The circumferential dimensions of hybrid rings can be changed in discrete steps without altering any of the circuit characteristics, by the application of either or both of the following rules.

(a) An integral number of wavelengths may be added to or subtracted from any arc (i.e. portion of ring between centers of adjacent connections).

(b) A pair of half wavelengths may be added to or subtracted from any two arcs.

These rules can be applied when it is desired to simplify the construction or to alter the structure to conform more readily to available space. Some of the dimensions in Figure 11 were arrived at using these rules. The proportions shown in Figure 10 conform in each case to the smallest ring in which all connections are separated by finite arcs. An additional pair of half wavelengths may be removed from each of the circuits of Figure 11 b and 11 e. Care must be taken, however, to preserve connections which are truly adjacent and which are not superposed symmetrically to the same point of the ring, if the properties of the original hybrid ring are to be preserved.

In general wave guide constructions are likely to demand an expansion from the dimensions of Figure 11. Rectangular wave

guides have become somewhat standardized upon an electric plane dimension only a little less than a half (free space) wavelength. Since the wavelength in the guide is seldom as great as two times the free space wavelength, the quarter (guide) wavelength spacings stipulated in Figure 11 correspond to extremely close spacings or sometimes even to overlapping of adjacent series connections. Greater difficulties will be experienced with adjacent connections in the magnetic plane. These troubles, when they arise, can be obviated by increasing the size of the ring according to the rules given. For example, the arcs in Figure 11 a may be increased to $3/4\lambda$, $3/4\lambda$, $3/4\lambda$, $1\ 1/4\lambda$.

It has been pointed out that for the wave guide form of the circuit in Figure 11a, the reactances associated with the junctions do not upset the performance because equal phase shifts are introduced as the waves travel past the two driven arms. The same arguments can be applied to the highly symmetrical circuits of Figure 11 d and 11 f. However, the situation is not so obvious with regard to the rings involving assymmetrically disposed mixed connections, since different phase shifts may be expected at series and shunt branches. Consider the circuit of Figure 11 b. When this is driven from the shunt arm or from the opposite series arm, equal phase shifts take place at the two identical, adjacent series arms and balance is secured, with the dimensions as given. If the junction reactances are reduced or eliminated by symmetrical tuning, balance and equal power division will still be retained.

General considerations based on reciprocity may now be cited to show that this same tuning automatically brings about balance, equal division of power, and impedance match when the circuit is driven from either of the opposed series arms, even though the connections adjacent to these arms have such a totally different character. The same arguments can be applied to Figure 11 e. The completely asymmetrical circuit of Figure 11 c remains, then as the only one whose dimensions may perhaps require substantial alteration when appreciable phase shifts are involved.

For a mathematical analysis see Appendix B.

Design Considerations: There are undesired reflections within the ring which arise in two ways:

a. Those at the tee junctions to the ring caused by the abrupt geometry which characterizes these junctions. These may be reduced as in the magic tee by introducing in the vicinity of the junction appropriate reactive elements.

b. Those whose source may be traced to the inherent resistive mismatch which results from using wave guides of approximately the same size for the ring and all the arms. In this case a generator of impedance K_0 is connected by two loads essentially in series, each of which has an impedance approximately equal to K_0. This mismatch can be readily eliminated by altering the characteristic impedance of one or more of the wave guides involved.

Frequency Characteristics: Of the various electrical properties the balance between opposite arms is the most critical one. Consider what happens when in Figure 9 there are sent into the ring waves of a frequency different from that originally considered. There will still be cancellation of fields at point 4, but now the spacing in the standing wave pattern is changed and the voltage maximum is displaced from point 3. An arm attached at 3 will thus not be completely uncoupled from the arm at 1. This reasoning can be applied to the circuits shown in Figures 11a, 11d, and 11f, and also to those pairs of opposite connections in Figures 11b and 11e, which are balanced by virtue of two paths

around the ring which differ by a half wave length; that is, to all cases in which the opposite connections are both parallel or both series. A different situation exists when balance is obtained between a series connection and a shunt connection located at geometrically opposite points across the rings, since the standing wave pattern at the point diametrically opposite the feed point does not shift with frequency. This argument applies to Figure 11c and one pair of opposed arms in Figures 11b and 11e. When a high degree of balance is desired between one particular pair of arms over as broad a band as possible, the circuits of Figures 11b and 11e will be preferred, for they offer driven arms which are symmetrically connected.

If the loads attached to the arms of a hybrid ring are maintained nearly reflectionless over a band of frequencies, and if the junction reactances in the ring are effectively cancelled throughout this band, important variations in the power division or in the resistive matching ability of the network will not be expected. In general, limits to the usable band width will be set by deviations from balance or by inability to tune out satisfactorily the internal reflections associated with the junctions.

APPLICATIONS

The very nature of a hybrid circuit is such as to suggest immediately numerous applications in duplexing. In Figure 2, for example, the following connections may be made: at A, a transmitter; at B, a transmission line or antenna; at C, a receiver; at D, a dummy load. Of the power sent into the network from the transmitter, half will be sent out the line or antenna, half will be dissipated in the dummy load, and none will appear in the receiver. Signals entering the network from the line or antenna will be divided equally between the receiver and transmitter. There will therefore be a 3-decibel loss in reception and in transmission, or the total round trip loss of 6 decibels often associated with hybrid devices. This duplex system still compares favorably with the two simplex systems necessary to accomplish the same result, for the total area of two similar antennas associated with the simplex systems may be devoted to a single antenna with a 3 decibel increase in gain, or in the case of transmission lines, the two lines needed in simplex operations may be replaced by a single improved line, so that again the 6 decibel round-trip loss is recovered. This loss can also be reduced materially by the appropriate employment of active circuit elements, such as gas-discharge switches.

Another important field of application occurs in connection with balanced mixers and converters. As an interesting preliminary to this subject, consider the following argument. In

Figure 2 let the terminals B and D be joined externally at a junction E. This junction may be so located that when power is introduced at A, the equal amounts of power delivered to B and D reach E in phase. All of the power from A will therefore be developed in the appropriate load at E. Since A and E are completely coupled, the terminals C, by reciprocity, have no connection with either A or E, and hence play no part in the circuit. Thus, it is not possible to combine the two radio-frequency voltages to drive a single load and to avoid the three decibel loss accompanying the use of a dummy load, without at the same time destroying the hybrid properties of the circuit.

It is possible however to avoid a loss, when detection is involved, by combining detected signals. It is this principle which is involved in the use of a hybrid network for a balanced converter in the manner represented schematically in Figure 12. The intermediate frequency voltages appearing at B' and D' must be combined with due regard for their relative phase. While it is important that there is no loss in signal power over that normally associated with detection, this in itself is scarcely a reason for preferring the balanced converter. Its advantages however are numerous including the isolation between the signal and and local beating oscillator and the elimination of noise contributions from the local oscillator. This has indeed been an application in which these hybrid circuits have been extensively used in microwave systems. For this purpose the hybrid junction or

magic tee has been chosen, for reasons of compactness, convenient geometry, and broad frequency response.

One of the more important uses of these hybrid circuits in impedance bridges may be exemplified by the following method of measurement of the band pass characteristic of low-QTR tubes. Assume that a magic tee is used and arrange it as follows: Test piece or unknown impedance Z_x on arm 1, generator on arm 3, detector on arm 4, and a characteristic impedance Z_0 on Arm 2. The power input arm 4 is a measure of the magnitude of the voltage reflection coefficient of the unknown impedance Z_x. The error encountered depends on the match of the detector and generator and the mechanical asymmetry in the magic tee. If this tee is used with a modulated signal source and a crystal detector, the tuning of a device which must be matched at one frequency or which must have a reflection coefficient less than a certain value at the one frequency is simple. The tuning of the device is adjusted for minimum power in the output arm. With a perfect magic tee this will be zero for match; otherwise, it will depend on the desired reflection coefficients.

With appropriate precautions it is possible to use a single magic tee with several input frequencies. By the use of a corresponding number of local oscillators in a circuit similar to a spectrum analyzer, the power from the output arm may be displayed on an oscilloscope in the form of pips, one pip for each frequency. If three frequencies are used, the behavior of the reflecting element

at the midband and band-edge frequencies is easily determined, and may be observed at a glance. If the device is calibrated with a reflection of known magnitude, quantitative data may be obtained. For example, if it is desired to check a resonant window or a single resonant element, such as is used in a band pass TR tube, the three frequencies corresponding to midband and the two bandedge frequencies are set on the bridge. The element under test, backed by a matched load, is put on one arm of the magic tee. The heights of the pips then give the desired information. The tuning of an element can be accomplished by adjusting for a symmetrical pattern. In addition to a quick examination of the characteristics of single elements, it is also possible to tune two-element bandpass TR tubes and to check the overall response curve, that is, the standing wave ratio looking through the tube into a matched load at three different points in the passband.

This type of bridge is excellent for production checking of components, such as the windows of the bandpass TR tubes, especially for tuning, since the necessary symmetry of the Q-curve for the proper frequencies can be noted at a glance. Any necessary changes on transmission-resonant elements, such as grinding the glass in the windows or filing metal in other types of elements, may be quickly checked between steps by noting the change in the pips.

Beyond these and other applications for hybrid circuits in microwave systems, there are numerous uses in laboratory measure-

ments. These will arise whenever it is necessary to establish directional balance or to make special use of the phase properties which characterize these circuits.

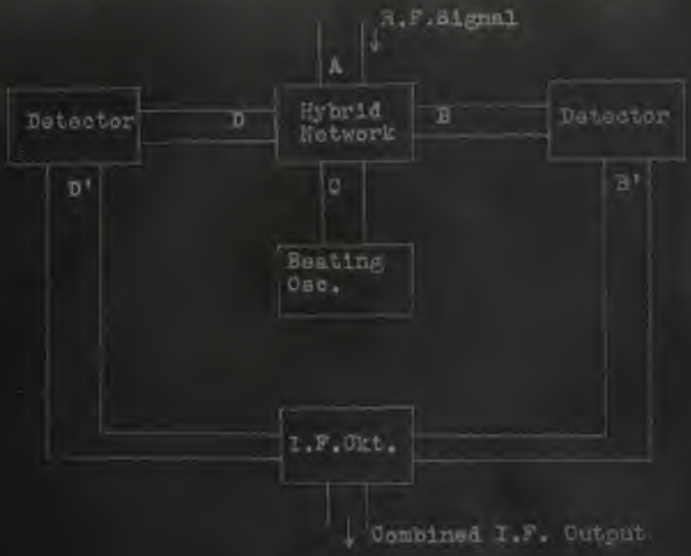

R.F.Signal
A
Detector
D
Hybrid Network
B
Detector
D'
C
B'
Beating Osc.
I.F.Ckt.
Combined I.F. Output

FIG.12

FIG. 13

SIX-ARM MICROWAVE BRIDGE

A six-arm wave guide structure shown in Figure 13 has been developed which is a true microwave equivalent of a wheatstone bridge. The other microwave devices which have been discussed and which measure impedances by comparison are not actually bridges in the sense that one uses the term at low frequencies. To be a true bridge of the wheatstone type, they must obey the equation

$$AB = CD$$

$$(1) \text{ or } B = \frac{C}{A} D$$

Where A, B, C, and D are impedances, three of which are known and the other unknown. The magic tee and retraces are not true bridges, for theyobey the equation

$$(2) \quad A = B$$

It is necessary in (2) to provide an adjustable impedance equal exactly to the unknown. Thus, the standard impedance will require both a variable resistance and a variable reactance. At microwave frequencies a variable resistance is difficult to provide. This is a disadvantage of using a hybrid ring or junction. With the true wheatstone bridge, however, any impedance may be matched by using a fixed resistance and variable reactances. Both of these are simple to make at microwave frequencies. As an example, let B be the unknown. Then if A and C are variable reactances consisting of shorted sections of transmission lines of variable length,

the ratio C/A can be adjusted to have any real value desired. D. consists of a perfectly terminated line in series with a variable reactance, which is another shorted-section line. By adjusting the latter, D can be made to have any arbitrary phase angle. Therefore, D can be adjusted to have the phase angle of the unknown and the ratio C/A can be adjusted to give the correct magnitude. The value of the unknown B, then, is determined from the physical lengths of stubs A, C, and D.

The bridge shown in Figure 13 satisfies 1. There are six terminal pairs; the four impedance arms, and an input and output arm. This bridge is made by joining six pieces of rectangular wave guides at a symmetrical junction. Opposite pairs of arms, such as A and C or B and D, lie on common axes, but are rotated 90° with respect to each other.

This bridge was first suggested by R. Dicke formerly of the M. I. T. Radiation Laboratory. It appeared plausible that the device in Figure 13 was a true Wheatstone bridge from consideration of the geometry and the way in which the various guides are connected. The designer then proceeded to show analytically that this device is the equivalent circuit.

The procedure was to select sets of waves in the structure, such that the field distribution due to any arrangement of terminating admittances and sources can be represented by linear combination of these waves. Thus, upon consideration of these waves and the symmetry properties of the bridge, relations between

the input admittance at one terminal pair and all the other five terminating admittances can be found. The waves that are set up are for the lowest mode in the wave guide only; that is, it is assumed that one is measuring the field sufficiently far from the junction of the guides so that all the higher modes which are present in the fringing fields at the junction are attenuated. The effect of the junction appears in the equation for the bridge in the form of equivalent lumped susceptances at suitably chosen reference planes in the guides. The expression for the admittance relation among the six arms is:

$$3 - Y_x Y_{x'} Y_z + Y_x Y_{x'} Y_z + Y_x Y_{x'} Y_y + Y_x Y_{x'} Y_y + Y_y Y_{y'} Y_z + Y_y Y_{y'} Y_z + Y_y Y_{y'} Y_x + Y_y Y_{y'} Y_{x'} + Y_z Y_{z'} Y_x + Y_z Y_{z'} Yx' + Y_z Y_{z'} Y_y + Y_z Y_{z'} Y_{y'} + Y_x Y_y Y_z + Y_{x'} Y_{y'} Y_z + Y_x Y_{y'} Y_{z'} + Y_{x'} Y_y Y_{z'} = 0$$

where $Y_x = Y_x + jb$

$Y_{x'} = Y_{x'} + jb$ etc.

Y_x is the terminating admittance on arm x etc. This is the equation of a Wheatstone bridge with fringing susceptance of magnitude jb across each terminal. For the derivation of 3 see Bibliography No. 5.

In terms of this general admittance relation, the condition for a balanced bridge is that the input admittance is independent of the detector admittance, and a relation of the form 1 will fulfill this requirement. This balance equation, as modified by the fringing susceptances and using a slightly different notation,

can be written

$4 - (Y_a + jb)(Y_b + jb) = (Y_c + jb)(Y_d + jb)$

The equivalent circuit is shown in Figure 15.

Design Considerations and Bridge Characteristics: The ends of the wave guides which come together at the junction have triangular tongues, as shown in Figure 14, cut in each side, and when the six guides are placed together, these tongues mate to give the correct orientation for all the guides.

As is always the case for such equivalent circuits to be applicable, the admittances of impedances must be referred to certain special planes in the various arms of the structure. In this case, the equivalent plane for any of the bridge arms is the position of a shorting plunger which will cause a short to appear in the corresponding branch of the equivalent circuit. It is clear that, if the admittances Y_a, etc., which are measured at such equivalent planes satisfy an equation of the form 4, they will no longer satisfy it after they have been transformed an arbitrary distance down the line from the place at which they were originally defined. These equivalent planes can be located by moving shorts in two adjacent arms such as A and C, until the standing-wave ratio in the input circuit is infinite. In this case, the positions of the shorts determine the equivalent planes in A and C. The position of the node in the standing wave in the input arm also defines the equivalent plane in that arm.

There is a more practical method for locating the equivalent plane which depends on the fact that when shorts are located at the equivalent planes of A and C, corresponding to infinite admittances in the branch A-C, then the output of the detector

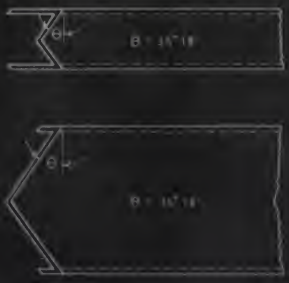


FIG. 14

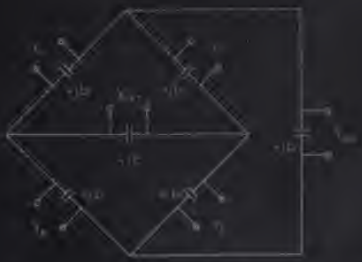


FIG. 15

in zero, independent of vaiations in Y_b and Y_d. The admittances Y_b and Y_d can be modulated by setting rotating electric fans near the open end of arms B and D, resulting in a modulated detector output. The short positions in A and C are then varied until the detector output is reduced to zero, which determines the equivalent planes for these arms, and also in the input arm. Planes in the other arms are located in the same way, by choosing different input arms.

In use as a bridge a calibrated movable short is placed on arm D in such a position that normalized admittance 1 + jY appears at the equivalent plane. Movable shorts are attached to arms A and C and the admittance to be measured is attached to arm B. Then

$$Y_b = \frac{Y_a}{Y_c} Y_d \text{ if fringing}$$

susceptance due to asymmetries in construction has been eliminated. This fringing susceptance is frequency-dependent but can be eliminated at any particular frequency by inserting a capacitive screw at the position of the equivalent plane or a quarter wavelength away, depending on the sign of jb.

Once properly adjusted, the bridge is simple to use, and measurements can be made rapidly. The unknown impedance is determined in terms of three physical lengths, the distances of the shorts from this correct reference planes, which is a very simple standard and is independent of detector calibration. The accuracy of the measurement is equivalent to that of a good standing-wave

et une conséquence le théorème de Bolzano-Weierstrass appliqué aux fonctions continues. Une méthode simple (mais pas toujours judicieuse) consiste à montrer que la fonction considérée est bornée et atteint sa borne supérieure.

Deux fonctions différentes ne peuvent pas avoir le même jeu de valeurs.

Soient f et g deux fonctions définies sur $\mathbb{R}$ telles que $f \circ g = g \circ f$. Une récurrence montre que $f^n \circ g = g \circ f^n$ pour tout $n \in \mathbb{N}$. Si f est bijective, l'application $f^{-1} \circ g \circ f$ est identique à g et a donc le même jeu de valeurs que g.

Si f est une fonction continue de $\mathbb{C}$ dans $\mathbb{C}$ et α un nombre complexe, on note $E_\alpha(f)$ l'ensemble des $z \in \mathbb{C}$ tels que $f(z) = \alpha$. Cet ensemble est fermé. Si f est non constante, $E_\alpha(f)$ est discret pour tout α dans $\mathbb{C}$ sauf éventuellement pour une valeur α_0. Si f est de plus entière, $E_\alpha(f)$ est infini pour tout α dans $\mathbb{C}$ sauf éventuellement pour une valeur α_0. Une fonction entière non constante prend donc presque toutes les valeurs de $\mathbb{C}$ une infinité de fois.

detector. The use of the three physical lengths to specify the unknown impedance can be facilitated with the aid of simple formulas or charts. Although, in the current version of the bridge described here, the fringing susceptance is eliminated for only one frequency, it should be possible by conventional broad banding techniques to make the device quite broad band, so that the fringing susceptances are almost zero over a considerable range of frequency.

Four-Terminal Application: Since the six-arm bridge is the exact equivalent of a Wheatstone bridge, it can be used as a four terminal lattice section in microwave filter design or in similar applications. Most microwave filter structures used up to now have been the equivalents of ladder sections. A lattice section is the most general form of four-terminal network and offers much greater flexibility in design .

As an example, the transfer constant and image impedances can be separately adjusted. Also, any physical four-terminal network can be synthesized by means of lattice sections, while this is not always true for ladder sections.

CONCLUSION

The magic tee, the directional coupler, and the hybrid ring may all be used in microwave bridges. However, the magic tee is chosen in most applications, principally, for reasons of compactness, convenient geometry, and broad frequency response.

The magic tee has the advantage of balance on the directional coupler since in the tee balance depends on geometrical symmetry alone. The directional coupler does, however, permit measurements to be made on a unit while in service.

The band width over which the hybrid ring operates differentially is inherently narrower than that of the magic tee. The rings do have considerably higher power capacity though.

The six-arm microwave bridge is an improvement for laboratory measurements. With this bridge it is possible to measure any impedance to about the same accuracy as with a standing-wave detector. A valuable feature of this instrument is that the standard impedances required are three variable reactances (movable shorting plungers) and a Z_0 termination. The data are obtained in the form of three lengths, the positions of the movable shorts.

The disadvantages of this Wheatstone equivalent bridge is narrow band width, and difficulties in construction, resulting in asymmetries and requiring tuning devices.

There are many applications for these bridges including production checking of bandpass characteristics, and in mixers, in converters, and in duplexers.

APPENDIX A

Analysis of Tee's

The following analysis uses the symmetry properties of solutions of Maxwell's equations in deriving results. These results are expressed in terms of a small number of parameters, which could be determined theoretically from an exact solution of Maxwell's equations, or which may be regarded as parameters in equivalent circuits for the various systems, which can be determined by appropriate experiments. The general method is to set up certain standing-wave solutions of Maxwell's equations, and to express the general solution as a linear combination of these independent standing waves.

If Maxwell's equations inside the cavity of the tee are solved, for a given frequency, an infinite variety of solutions are found, unless boundary conditions at the outputs are specified. If only those solutions in which there is real propagation in at least some output are considered, we are able to write all such solutions as linear combinations of a finite number of solutions. In the case of the tee, there are four such solutions.

Each of these solutions will have complicated behavior in the central part of the cavity, which we will consider only as to its symmetry. But in the wave guide, however, the field, which can be written as a superposition of all normal modes of the guide, becomes simplified by the attenuation of all those modes which are not propagated. We shall assume that the frequency is

such that only the dominant mode is propagated in the output guides; thus at some distance the field reduces to this mode, and can be characterized entirely by two constants, a phase and an amplitude, or a current and a voltage.

Now referring to Figure 3, let us choose symmetrically placed planes in arms 1 and 2 in an arbitrary manner. Four standing waves are then set up, two antisymmetric, two symmetric. One of each will have $V = 0$ at the planes of reference in 1 and 2, and one will have $i = 0$ at the planes. Now let us choose the plane of reference in 3 so that the antisymmetric function which has zero voltage across the reference planes in 1 and 2 likewise has zero voltage across the reference plane in 3. Then, let us choose the plane of reference in 4 so that the symmetric function which has zero voltage across the planes in 1 and 2 has zero voltage across the plane in 4. Thus, we have the following:

1. $V_{1a} = V_{2a} = V_{3a} = V_{4a} = 0$; $i_{1a} = -i_{2a}$; $i_{4a} = 0$

2. $V_{1b} = -V_{2b}$; $V_{4b} = 0$; $i_{1b} = i_{2b} = i_{4b} = 0$

3. $V_{1c} = V_{2c} = V_{3c} = V_{4c} = 0$; $i_{1c} = i_{2c}$; $i_{3c} = 0$

4. $V_{1d} = V_{2d}$; $V_{3d} = 0$; $i_{1d} = i_{2d} = i_{3d} = 0$.

The voltages and currents at the various reference planes are then given by:

5. $V_1 = C_b V_{1b} + C_d V_{1d}$; $V_2 = -C_b V_{1b} + C_d V_{1d}$

6. $V_3 = C_b V_{3b}$; $V_4 = C_d V_{4d}$

7. $i_1 = C_a\, i_{1a} + C_c i_{1c}$; $i_2 = -C_a i_{1a} + C_c i_{1c}$

8. $i_3 = C_a i_{3a} + C_b i_{3b}$; $i_4 = C_c i_{4c} + C_d i_{4d}$

Where C_a, C_b, C_c, and C_d are appropriate complex coefficients used when building up general solutions by superposing solutions.

From the above, we can set up two equations.

9. $$\frac{i_3}{V_3} = \frac{i_{3b}}{V_{3b}} + \frac{C_a\, i_{3a}}{C_b\, V_{3b}} = \frac{i_{3b}}{V_{3b}} + \frac{i_{3a}\, V_{1b}}{V_{3b}\, i_{1a}}\left(\frac{i_1 - i_2}{V_1 - V_2}\right)$$

10. $$\frac{i_4}{V_4} = \frac{i_{4d}}{V_{4d}} + \frac{C_c\, i_{4c}}{C_d\, V_{4d}} = \frac{i_{4d}}{V_{4d}} + \frac{i_{4c}\, V_{1d}}{V_{4d}\, i_{1c}}\left(\frac{i_1 + i_2}{V_1 + V_2}\right)$$

Let

11. $Y_3 = Z_o\,(i_3/V_3)$

12. $Y_4 = Z_o\,(i_4/V_4)$

13. $Y_1 = Z_o\,(i_1/V_1)$

14. $Y_2 = Z_o\,(i_2/V_2)$

This leads to

15. $$Y_3 = -jb_3 - a_3\left(\frac{Y_1V_1 - Y_2V_2}{V_1 - V_2}\right)$$

16. $$Y_4 = -jb_4 - a_4\left(\frac{Y_1V_1 + Y_2V_2}{V_1 + V_2}\right)$$

Where

17. $-jb_3 = \frac{i_{3b}}{V_{3b}} Z_o$; $-jb_4 = \frac{i_{4d}}{V_{4d}} Z_o$

18. $a_3 = -\frac{i_{3a}}{V_{3b}}\frac{V_{1b}}{i_{1a}}$ $\quad a_4 = \frac{-i_{4c}}{V_{4d}}\frac{V_{1d}}{i_{1c}}$

From 15 and 16

$$19.\quad Y_1Y_2 + \frac{(Y_3 + jb_3)(Y_4 + jb_4)}{a_3 a_4} + \frac{1}{2}(Y_1 + Y_2)\left[\frac{(Y_3 + jb_3)}{a_3} + \frac{(Y_4 + jb_4)}{a_4}\right] = 0$$

This is one form of the admittance equation for the T.

In 19 if $Y_1 = Y_2$ we have

$$20.\quad \left(Y_1 + \frac{Y_3 + jb_3}{a_3}\right)\left(Y_1 + \frac{Y_4 + jb_4}{a_4}\right) = 0$$

One or the other factor must be zero. Thus, suppose the first factor is zero; we then have for the admittance seen looking into the output 3

$$21.\quad -Y_3 = jb_3 + a_3Y_1$$

Thus, the admittance is independent of Y_4. The reason is that with equal loads in 1 and 2, no power fed into output 3 proceeds to 4; it is all divided equally between 1 and 2. Hence, there is no effect of the admittance in 4 on the admittance as seen in 3. Similarly, if the power is fed into 4, none will proceed into 3, and we have

$$22.\quad -Y_4 = jb_4 + a_4Y_1$$

These results suggest an experimental method of determining the constants of the T. Remembering that any symmetrically chosen pair of planes in one and two may be taken as planes of reference in those outputs, choose them, and then place short-circuiting plungers at these planes, so that $Y_1 = Y_2 = \infty$. Then feed power into output 3. From 21 we should have $-Y_3 = \infty$ or an infinite standing wave ratio, with the plane of reference in 3 at the standing wave minimum. Then remove the plungers and

match 1 and 2, so that $Y_1 = Y_2 = 1$. Now feed power into 3 and observe the admittance $-Y_3 = jb_3 = 2a_3$, from which b_3 and a_3 can be found by standing wave measurement. By similar methods in output 4, b_4 and a_4 are found.

Now to consider the question of power flow, let P be the power flow out an output and b' be the real part of Y. Then the following is true

23. $P_1 : P_2 : P_3 : P_4 = b_1'/|V_1|^2 : b_2'/|V_2|^2 : b_3'/|V_3|^2 : b_4'/|V_4|^2$

From 6, 15, and 16

24. $V_3 = \frac{V_{3b}}{V_{1b}}\left(\frac{V_1 - V_2}{2}\right)$; $V_4 = \frac{V_{4d}}{V_{1d}}\left(\frac{V_1 + V_2}{2}\right)$

The only non-trivial relations of $\frac{V_{3b}}{V_{1b}}$ and $\frac{V_{4d}}{V_{1d}}$ are

25. $\frac{V_{3b}}{V_{1b}} = \sqrt{\frac{2}{a_3}}$; $\frac{V_{4d}}{V_{1d}} = \sqrt{\frac{2}{a_4}}$

From 23, 24, and 25

26. $P_1 : P_2 : P_3 : P_4 = b_1' \left|\frac{Y_3 + jb_3}{a_3} + Y_2\right|^2 : b_2' \left|\frac{Y_3 + jb_3}{a_3} + Y_1\right|^2 :$

$\frac{b_3'}{2a_3}|Y_2 - Y_1|^2 : \frac{b_4'}{2a_4}\left|2\left(\frac{Y_3 + jb_3}{a_3}\right) + Y_1 + Y_2\right|^2$

27. $P_1 : P_2 : P_3 : P_4 = b_1' \left|\frac{Y_4 + jb_4}{a_4} + Y_2\right|^2 : b_2' \left|\frac{Y_4 + jb_4}{a_4} + Y_1\right|^2 :$

$\frac{b_3'}{2a_3}\left|2\left(\frac{Y_4 + jb_4}{a_4}\right) + Y_1 + Y_2\right|^2 : \frac{b_4'}{2a_4}|Y_2 - Y_1|^2$

Using 19 two other equations of this form may be established, all equivalent to each other. We will use 27 for the case where Y_1

is nearly equal to Y_2 and power is being fed into the output 3. This is approximately the situation described in 21. For $Y_1 = Y_2$ equation 27 reduces to

28. $P_1 : P_2 : P_3 : P_4 = b_1' \left| \frac{Y_4 + jb_4}{a_4} + Y_1 \right|^2 : b_1' \left| \frac{Y_4 + jb_4}{a_4} + Y_1 \right|^2 : \frac{2b_3'}{a_3} \left| \frac{Y_4 + jb_4}{a_4} + Y_1 \right|^2 : 0$

Taking the real part of 21, we have in this case $-b_3' = a_3 b_1'$. Thus in this case

29. $P_1 : P_2 : P_3 : P_4 = 1 : 1 : -2 : 0$ or the power flowing into output 3 is equally divided between 1 and 2, none flowing out 4, as we have mentioned earlier. If Y_1 is not exactly equal to Y_2, however, 27 serves to give the power flowing out of output 4, and to show that it is proportional to the square of the magnitude of $Y_2 - Y_1$.

Equation 26 is useful if Y_1 is nearly equal to Y_2, but power is fed into output 4.

In the equations derived above, there is not complete symmetry between the outputs 1 and 2 on the one hand, 3 and 4 on the other, the admittances Y_1 and Y_2 are balanced by the more complicated quantities $\frac{Y_3 + jb_3}{a_3}$ and $\frac{Y_4 + jb_4}{a_4}$. In order to secure complete symmetry, we must have $b_3 = b_4 = 0$, $a_3 = a_4 = 1$. This can be done by inserting irises in outputs 3 and 4. The balancing quantities above will then be replaced by Y_3 and Y_4 respectively in 19, 26, and 27, and there will be complete symmetry between

the two sets of outputs. Equation 21 will then become $-Y_3 = Y_1$, showing that if $Y_1 = Y_2 = 1$, so that the two terminals 1 and 2 are matched, we shall see a match looking into output 3. A Tee with these characteristics is generally called a Magic Tee.

APPENDIX B

Analysis of Hybrid Rings

This analysis is on hybrid rings of the series type. The ring is considered as a continuous line closed on itself. The sections between series taps are then treated as being made up of integral single or multiple quarter wave line sections. Lattice or equivalent T sections are derived for 1, 2, 3, and 4 quarter wavelength sections, ignoring line dissipation. Then, equivalent mesh networks are drawn from these equivalences. The mesh networks are then solved by determinants. To study some effects of frequency shift off the design center, where the mean periphery of the ring departs from an exact integral number of quarter wavelengths, the increments in the element values for a quarter-wave equivalent T section are calculated and utilised. The example studied is a ring of 1 1/2 λ mean perimeter with three and four taps.

In this analysis Z_0 represents guide impedance and Z_r represents annulus impedance. These rings may be considered on the basis that the annular slot is a transmission line, of characteristic impedance Z_r and propagation constant P. This transmission line is closed upon itself. Series connections are made by the wave guide connections. The outlets of the wave guide are assumed to present wave guide characteristic impedance to the ring of Z_0. This may be shown as in Figure 16.

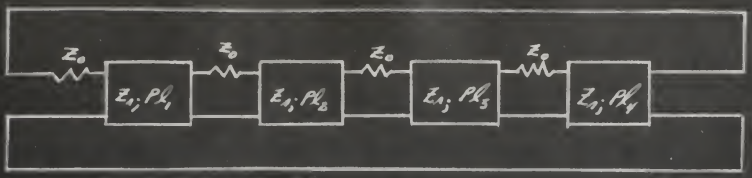


FIG 16

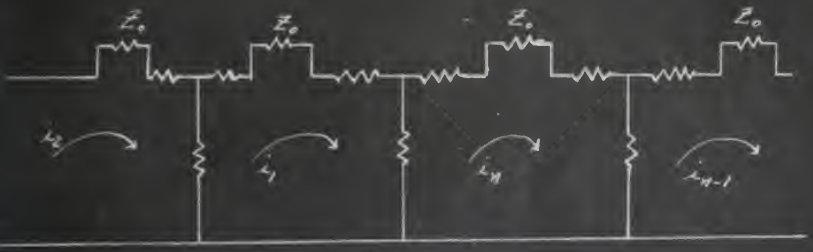
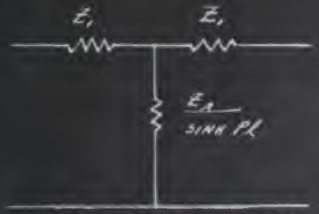
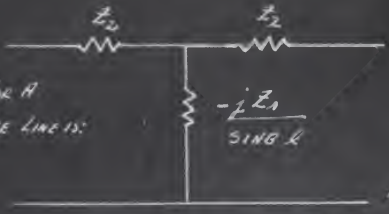


FIG 17



$Z_1 = \tanh \frac{\beta l}{2}$

WHICH FOR A LOSS-FREE LINE IS:



$Z_2 = j Z_1 \tan \frac{\beta l}{2}$

FIG 18

Now consider the line as made up of the appropriate number of quarter-wave sections, with series taps as shown in Figure 17.

The shunt impedances are identical; call each X. The series impedances are made identical by first assuming a tap at each quarter-wave junction. Let each series leg be S. Then, for a 2 1/2 wave length ring or N = 10, the (skew symmetrical) circuit determinant is:

(1) $D_{10} =$

$$\begin{vmatrix}
S+2x & -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X \\
-X & S+2x & -X & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & -X & S+2x & -X & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & -X & S+2x & -X & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & -X & s+2x & -X & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & -X & S+2x & -X & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & -X & S+2x & -X & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & -X & S+2x & -X & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & -X & S+2x & -X \\
-X & 0 & 0 & 0 & 0 & 0 & 0 & 0 & -X & S+2x
\end{vmatrix}$$

For the case of an exact number of quarter wave lengths around the ring, all $X_{1-n} = jZ_r$ and all $S_{1-n} = Z_o + 2jZ_r$.

Then, the determinant becomes

$$\begin{vmatrix} \cos\alpha & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 2\cos\alpha & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 2\cos\alpha & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 2\cos\alpha & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 2\cos\alpha & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2\cos\alpha & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 2\cos\alpha \end{vmatrix}$$

$$= \frac{\sin(n+1)\alpha}{\sin\alpha}$$

$$\begin{vmatrix} \cos\alpha & 1 & 0 & 0 & \cdots & 0 & 0 & 0 \\ 1 & 2\cos\alpha & 1 & 0 & \cdots & 0 & 0 & 0 \\ 0 & 1 & 2\cos\alpha & 1 & \cdots & 0 & 0 & 0 \\ 0 & 0 & 1 & 2\cos\alpha & \cdots & 0 & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & \cdots & 2\cos\alpha & 1 & 0 \\ 0 & 0 & 0 & 0 & \cdots & 1 & 2\cos\alpha & 1 \\ 0 & 0 & 0 & 0 & \cdots & 0 & 1 & 2\cos\alpha \end{vmatrix}$$

$$= \frac{\sin(n+1)\alpha}{\sin\alpha}$$

$$(2)\quad D'_{10} = \begin{vmatrix} Z_o & jZ_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & jZ_r & E_1 \\ jZ_r & Z_o & jZ_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & E_2 \\ 0 & jZ_r & Z_o & jZ_r & 0 & 0 & 0 & 0 & 0 & 0 & E_3 \\ 0 & 0 & jZ_r & Z_o & jZ_r & 0 & 0 & 0 & 0 & 0 & E_4 \\ 0 & 0 & 0 & jZ_r & Z_o & jZ_r & 0 & 0 & 0 & 0 & E_5 \\ 0 & 0 & 0 & 0 & jZ_r & Z_o & jZ_r & 0 & 0 & 0 & E_6 \\ 0 & 0 & 0 & 0 & 0 & jZ_r & Z_o & jZ_r & 0 & 0 & E_7 \\ 0 & 0 & 0 & 0 & 0 & 0 & jZ_r & Z_o & jZ_r & 0 & E_8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & jZ_r & Z_o & jZ_r & E_9 \\ jZ_r & 0 & 0 & 0 & 0 & 0 & 0 & 0 & jZ_r & Z_o & E_{10} \end{vmatrix}$$

For a 1 1/2 λ ring n = 6 and the system becomes

$$(3)\quad D_6 = \begin{vmatrix} Z_o & jZ_r & 0 & 0 & 0 & +jZ_r \\ +jZ_r & Z_o & +jZ_r & 0 & 0 & 0 \\ 0 & +jZ_r & Z_o & +jZ_r & 0 & 0 \\ 0 & 0 & +jZ_r & Z_o & +jZ_r & 0 \\ 0 & 0 & 0 & +jZ_r & Z_o & +jZ_r \\ +jZ_r & 0 & 0 & 0 & +jZ_r & Z_o \end{vmatrix}$$

For the study of the effects occurring if we move off the design center, the individual T's may be changed to a length $l = \frac{\lambda}{4} \pm \frac{\lambda}{N}$. Assuming there is no line dissipation and N large so $\frac{\lambda}{N} << \lambda$, then each series arm is:

$$(4)\quad j Z_1 \tan\left[\frac{2\pi}{\lambda}\left(\frac{l}{2}\right)\right] = j Z_1 \tan\left[\frac{2\pi}{2\lambda}\left(\frac{\lambda}{4} \pm \frac{\lambda}{N}\right)\right] =$$

$$j Z_1 \tan\left[\frac{\pi}{4} \pm \frac{\pi}{N}\right] = j Z_1 \quad \frac{1 \pm \tan\frac{\pi}{N}}{1 \mp \tan\frac{\pi}{N}} =$$

$$\begin{vmatrix} & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \end{vmatrix} =$$

$$\begin{vmatrix} & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \\ & & & & & & & & & \end{vmatrix} =$$

$$j Z_1 \left(1 \pm \frac{2\pi}{N}\right) = j Z_1 (1 \pm \Delta)$$

In a similar manner, each shunt arm is:

(5) $$\frac{Z_1}{j \sin\left[\frac{2\pi}{\lambda} \ell\right]} = \frac{Z_1}{j \sin\left[\frac{2\pi}{\lambda}\left(\frac{\ell}{4} \pm \frac{\lambda}{N}\right)\right]} = \frac{Z_1}{j \sin\left[\frac{\pi}{2} \pm \frac{2\pi}{N}\right]}$$

$$= \frac{-j Z_1}{\sin\frac{\pi}{2} \cos\frac{2\pi}{N}} = \frac{-j Z_1}{\cos\Delta} \doteq \frac{-j Z_1}{1 - \frac{\Delta^2}{2}} \doteq$$

$$-j Z_1 \left(1 + \frac{\Delta^2}{2}\right) \doteq -j Z_1$$

Therefore, to a first approximation the shunt arm is not affected by a small shift off design center. The shunts X thus remain $-jZ_r$ and

(6) $S + 2X = Z_o + 2jZ_r \pm 2j\Delta Z_r - 2jZ_r = Z_o \pm 2j\Delta Z_r$

Therefore, the last two determinants, 2 and 3, can be used by merely considering $Z_o + 2\Delta Z_r$ as a special value of Z_o.

In 2 the driving voltages are added, opposite their associated meshes, in an external column at the right. This gives the current solutions. In the case being considered the number of driving voltages is usually one, never more than two; so the column will be zeros, except for one or two meshes. The current in any mesh is a fraction having D as denominator, and as numerator

the minor formed from D by substituting the e.m.f. column in the column corresponding to the mesh where the current is desired, i.e. column n if I_n is desired.

Since D is common to all mesh current expressions, questions of relative power division between branches or of null balance can be handled by operations performed entirely with the numerator minors.

Some slight advantage in evaluating the numerator minors is gained by proceeding where possible so as to make I_1 or I_n the desired current.

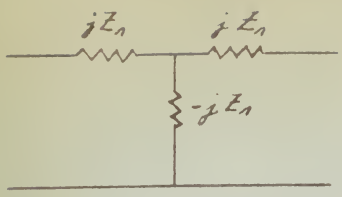
Alternatively, meshes where $Z_o = 0$ can be chosen. Another needed quantity is driving point impedance. Since $I_n = \frac{Ed_n}{D}$, then $\frac{1}{Z_{DP}} = \frac{dn}{D}$ and $Z_{DP} = \frac{D}{dn}$. The resulting impedance will include an extra Z_o, the generator impedance, to which we must match.

Another method of analysis is by using equivalents. The equivalent T for a recurrent structure of constants Z (characteristic impedance) and P propagation constant (where $P = A + jB$) per unit length is as shown in Figure 18.

Once the circuit is diagrammed using the above equivalences, it can be reduced to simpler form by successive combinations of T's, by well known formulae.

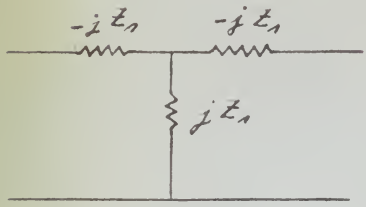
Now, we shall consider two special cases.

Case I. 1 1/2 λ Ring with three arms used as a power divider two way.



LENGTH = $\frac{\lambda}{4}$

LENGTH = $\frac{\lambda}{2}$
(T BECOMES INDETERMINATE)



LENGTH = $\frac{3}{4}\lambda$

LENGTH = λ

FIG. 19

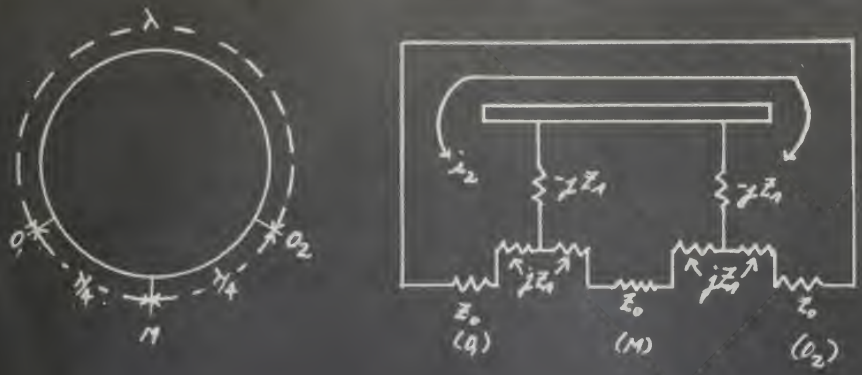


FIG 20

FIG 21

The method of equivalences will be used. The equivalent circuit is shown in Figure 20. From the figure it is seen that:

a. If power is fed in at M, it will divide equally between O_1 and O_2.

b. O_1 and O_2 are in proper wave length relationship for isolation relative to each other, but they are effectively in series and there will be no cancellation. Thus, the particular wavelength spacing is a necessary but not a sufficient condition.

c. If there is a voltage E at M, then we have

(7) $E = I_1 Z_o \qquad -I_2(2jZ_r)$

(8) $o = -I_1(-2jZ_r) + I_2(2Z_o)$ or

(9) $$\begin{vmatrix} E & Z_o & +2jZ_r \\ O & +2jZ_r & 2Z_o \end{vmatrix}$$ so

(10) $$\frac{I_1}{E} = \frac{2Z_o}{2Z_o^2 + 4Z_r^2}$$

The mesh impedance is then

(11) $$\frac{Z_o^2 + 2Z_r^2}{Z_o} = Z_o + \frac{2Z_r^2}{Z_o}$$

Therefore, an impedance match is secured if $2 Z_r = Z_o$

d. For a voltage e at O_1, the current at O_2 is obtained from

(12) $$\begin{vmatrix} e & 2Z_o & -2jZ_r \\ o & -2jZ_r & Z_o \end{vmatrix}$$

$$= \begin{vmatrix} \frac{\partial x}{\partial r} & \frac{\partial x}{\partial \theta} \\ \frac{\partial y}{\partial r} & \frac{\partial y}{\partial \theta} \end{vmatrix} \tag{2.2}$$

Thus

$$(13)\quad I_2 = \frac{eZ_o}{2Z_o^2 + 4Z_r^2}$$

For the impedance match condition of $\sqrt{2}Z_r = Z_o$, $I_2 = \frac{e}{4Z_o}$ which is one half the current which could be drawn through a load Z_o connected to a source Z_o with internal voltage e. Thus, there is a 6 db loss from O_1 to O_2.

Case II. 1 1/2 λ ring with four arms used as a power dividers two way.

The method of determinants will be used. See Figure 21. Let O_1 be in Mesh 1. Then D is in Mesh 4, O_2 is in Mesh 5 and M in Mesh 6. For meshes 2 and 3 $Z_o = 0$. Then the determinant 3 and its minor for mesh 5 with voltage applied at O_1, are respectively

$$D'_6 = \begin{vmatrix} Z_o & +jZ_r & 0 & 0 & 0 & jZ_r \\ jZ_r & 0 & jZ_r & 0 & 0 & 0 \\ 0 & jZ_r & 0 & jZ_r & 0 & 0 \\ 0 & 0 & jZ_r & Z_o & jZ_r & 0 \\ 0 & 0 & 0 & jZ_r & Z_o & jZ_r \\ jZ_r & 0 & 0 & 0 & jZ_r & Z_o \end{vmatrix}$$

and

$$d'_5 = \begin{vmatrix} jZ_r & 0 & jZ_r & 0 & 0 \\ 0 & jZ_r & 0 & jZ_r & 0 \\ 0 & 0 & jZ_r & Z_o & 0 \\ 0 & 0 & 0 & jZ_r & jZ_r \\ jZ_r & 0 & 0 & jZ_r & Z_o \end{vmatrix} = 0$$

Therefore, by adding outlet D, we have isolated branch O_1 from branch O_2. Since for this structure D is isolated from input at M, it must follow that an input at M will still divide equally between O_1 and O_2.

The band width over which a ring operates differentially is inherently narrower than that of a Tee; however, no matching stubs are required to achieve a fairly good standing wave ratio. The rings have considerably higher power capacity.

APPENDIX C

Analysis of the Direction Coupler as a Microwave Bridge

A directional coupler may be represented as an eight-terminal network similar to that shown in Figure 5 with the Tee replaced by the coupler. Let 2 and 4 be the secondary line and 1 and 3 the primary line. The amplitude transfer coefficient between the two lines is defined as

$$1.\quad K = \frac{E_{o2}}{E_{13}} = \frac{E_{o4}}{E_{11}}$$

when all lines are coupled to matched generators or loads. E_{13} is the voltage amplitude of the wave traveling in toward the coupler in arm 3, E_{o2} is the voltage amplitude of the wave traveling out from the coupler in arm 2, etc. The directive property of the coupler is

$$2.\quad d = \frac{E_{o4}}{E_{o2}}$$

when the generator is coupled to 3. The reflection coefficient of the slots is T_o; the matched termination or reference match, T_m; and the test piece or unknown impedance, T_t. If the generator and detector are matched to the line,

$$3.\quad E_{o2} = K(E_{13} + dE_{11})$$

also $$4.\quad E_{12} = T_m E_{o2}$$

Substitute 4 in 3

$$5.\quad E_{12} = T_m K(E_{13} + dE_{11})$$

This wave is incident in the direction of the detector, but is reduced by coupling to the primary line and reflection by the slots. Ignoring small secondary reflections and couplings, we have

$$6.\quad E_{04} = BE_{12} + KE_{11} + KdE_{13}$$

where $$7.\quad B = 1 - |K|^2 - |T_c|^2$$

for conservation of energy. Also

$$8.\quad E_{01} = BE_{13} + T_c E_{11}$$

and 9 $$9.\quad E_{11} = T_t E_{01}$$

Substitute 9 in 8

$$10.\quad E_{11} = \frac{T_t B E_{13}}{1 - T_c T_t}$$

From 6

$$11.\quad E_{04} = BT_m K E_{13} + \frac{BT_m K T_t (1-K) E_{13}}{1 - T_c T_t} + \frac{KT_t B E_{13}}{1 - T_c T_t} + KdE_{13}$$

The second term is small compared to the others and E_{04} may be approximated by

$$12.\quad E_{04} = KE_{13}\left(T_m B + \frac{T_t B}{1 - T_c T_t} + d\right)$$

Then the maximum and minimum values of the voltage to the detector are given by

$$13.\quad |E_{04}| = |K| \cdot |E_{13}| \left(\frac{|B| \cdot |T_t|}{|1 - T_c T_t|} \pm |B| \cdot |T_m| \cdot |T| \cdot |d|\right)$$

Since $|B|$ is very nearly unity, the phase error and imperfect reference match error enter in the same way as in the case of the

Tee. The factor d is the term in 13 which makes the directional coupler undesirable as a microwave bridge.

The directivity of a good directional coupler may be 30 to 35 db over a narrow band. Over a wide band, however, the directivity will be as low as 15 db for part of the band. This corresponds to /d/ >./ and the maximum error in measuring the voltage standing wave ratio of the test piece can be a factor of 1.22. A property of the directional coupler is that this error resulting from direct-coupling between the generator and the detector is frequency sensitive. In a Tee, an analagous error enters if the Tee is asymmetrical and there is cross-coupling between the series and parallel arms. This effect results only from asymmetry and is not frequency sensitive. Tee's may be constructed commercially so that the error from asymmetry does not exceed a factor of 1.01. This corresponds to a directivity of 52 db for a directional coupler.

BIBLIOGRAPHY

1. ANALYSIS AND PERFORMANCE OF WAVE GUIDE HYBRID RINGS FOR MICROWAVES; H. T. Budenbom; Bell Technical Journal, July, 1948.

2. THE THEORY AND EXPERIMENTAL BEHAVIOR OF RIGHT ANGLE JUNCTIONS IN RECTANGULAR SECTION WAVE GUIDES, I. E. E. Journal, Sept. 1946.

3. HYBRID CIRCUITS FOR MICROWAVES, W. A. Tynell, Proc. I. R. E. Nov. 1947.

4. TECHNIQUE OF MICROWAVE MEASUREMENTS, C. G. Montgomery, Radiation Laboratory Series #11, McGraw-Hill.

5. A MICROWAVE IMPEDANCE BRIDGE, H. Chodown, E. L. Ginzton, and F. Kane, Proc. I.R.E., June 1949.

6. BROAD-BAND POWER MEASURING METHODS AT MICROWAVE FREQUENCIES, L. E. Norton, Proc. I.R.E., July, 1949.

7. MICROWAVE PHASE FRONT MEASUREMENTS FOR OVERSEAS PATHS OF 12 and 32 MILES, A. W. Straiton, Proc, I. R. E., July 1949.

8. MICROWAVE MEASUREMENTS AND TEST EQUIPMENTS, F. J. Gaffney, Proc. I. R. E., October 1946.

9. MICROWAVE CONVERTERS, C. F. Edwards, Proc. I.R.E., Nov. 1947.

10. THE USE OF HYBRIDS, BOTH JUNCTIONS AND CIRCLES, AS APPLIED TO BOTH DUPLEXER AND MIXER DESIGN, E. G. Schneider, Proc, I.R.E. Aug. 1946.

11. A METHOD OF MEASURING PHASE AT MICROWAVE FREQUENCIES, R. D. Robertson, Bell System Technical Journal, Ja. 1949.

12. BRIDGE CIRCUITS FOR MICROWAVES, W. A. Tyrell, Bell Telephone Laboratories Report MM-42-160-10(BTL228)

13. MICROWAVE TRANSMISSION DESIGN DATA, Moreno, McGraw-Hill.

14. MICROWAVE DUPLEXERS, Ammillin and Montgomery, Radiation Lab. Series #14, McGraw-Hill.

15. MICROWAVE TRANSMISSION CIRCUITS, Rayan, Radiation Lab. Series #9, McGraw-Hill.

16. MICROWAVE MEASUREMENTS, Caffney, Proc. I.R.E., Oct. 1946.

17. THE USE OF THE MAGIC TEE MICROWAVE BRIDGE IN MEASURING IMPEDANCES, R. H. Kyle, M.I.T.-643.

18. MICROWAVE TEST EQUIPMENT, Polytechnic Research and Development Company.

19. MAGIC TEE WAVE GUIDE JUNCTION, G. Saxon and C.N. Miller, Wireless Engs., May 1946.

20. WAVEGUIDE HYBRIDS, W. A. Tynell, Bell Lab. Rec. Jan. 1948.

21. TRANSMISSION CIRCUITS FOR TELEPHONE COMMUNICATION, K. S. Johnson, D. Van Nostrand Co. N.Y.

22. COMMUNICATION NETWORKS, E. A. Guillemin, Vols. I and II, John Wiley and Sons, Inc.

23. A BROAD BAND BALANCED X-BAND CRYSTAL CONVERTER, W. W. Sharpless, Bell Tel. Lab. Report MM-44-160-134.

24. PRECISION MEASUREMENTS OF IMPEDANCE MISMATCHES IN WAVE GUIDES, A. F. Pomeroy, Bell System Tech. Journal, July 1947.

25. ELECTROMAGNETIC WAVES, S. A. Schelkunoff, D. Van Nostrand Co.

26. A WAVEGUIDE BRIDGE FOR MEASURING GAIN AT 4000 Mc, A. L. Samuel

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

[illegible]

and G. F. Crandell, Proc. I.R.E., Nov. 1948.

27. DIRECTIONAL COUPLERS, W. W. Mumford, Proc. I.R.E., February, 1947.

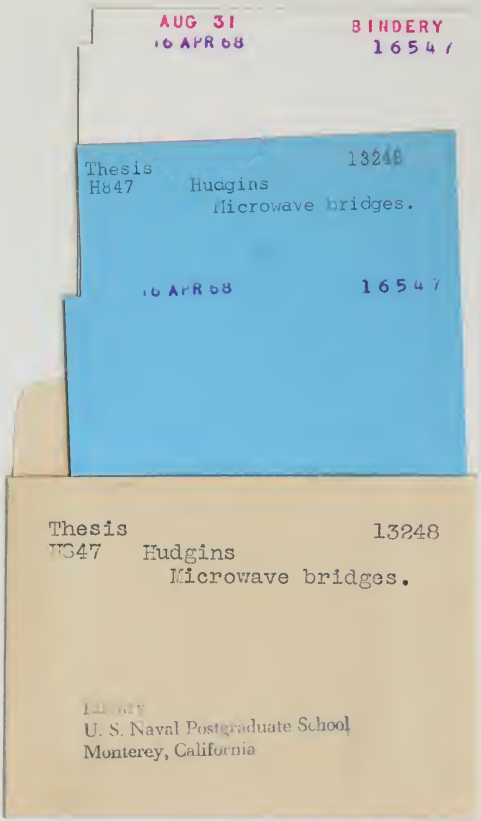
28. THE THEORY OF SYMMETRICAL WAVEGUIDE T'S, J. C. Slater; M.I.T Technical Report No. 37, April 3, 1947.

VITA

Vance Robert Hudgins was born August 31, 1919, at Spartanburg, S. C. He attended elementary and secondary schools in South Carolina, completing high school at Spartanburg, S. C. in 1935. He attended Wofford College for one year and then entered the United States Naval Academy. Upon completion of the course of instruction at that institution in 1941, and upon receiving a B. S. degree, he entered the United States Marine Corps.

Mr. Hudgins has served continuously in the Marine Corps as a commissioned officer, both in the line and as a naval aviator. He has performed various command and staff duties including that of Squadron Commanding Officer, Group Personnel Officer, and instructor at the Marine Corps Schools. He has served three overseas tours during the last war and received several medals as commendation for excellent performance of duties. His present rank is Major U. S. M. C.

He entered the Post Graduate School at Annapolis, Md. in 1947 to take a graduate course in "Guided Missile Guidance." After one year at this institution, he transferred to The Johns Hopkins University to continue his graduate work for two more years in the Department of Electrical Engineering.


AUG 31
16 APR 68
BINDERY
16547
Thesis
H847 Hudgins
Microwave bridges.
13248
16 APR 68
16547
Thesis
H847 Hudgins
Microwave bridges.
13248
U. S. Naval Postgraduate School
Monterey, California

ARTISAN
GOLD LETTERING
AND
SMITH BINDERY
OAKLAND, CALIF.

thesH847
Microwave bridges.
3 2768 001 03547 0
DUDLEY KNOX LIBRARY